(15) 1. a) Compute $\int_0^1 \int_{-y}^{1+\sqrt{y}} y\,dx\,dy$.

b) Write this iterated integral in $dy\,dx$ order. You may want to begin by sketching the area over which the double integral is evaluated. You are **not** asked to evaluate the $dy\,dx$ result, which may be one or more iterated integrals.

(14) 2. Compute the triple integral of $\dfrac{1}{(x^2+y^2+z^2)^2}$ over the region in $\mathbb{R}^3$ which is in the first octant ($x \geq 0$ and $y \geq 0$ and $z \geq 0$) <u>and</u> outside the unit sphere ($x^2+y^2+z^2=1$).

(12) 3. Find values of the constants A and B so that the vector field

$$\mathbf{V} = (x^3 + Ax^2y + y^3)\mathbf{i} + (Bxy^2 - 2x^3)\mathbf{j}$$

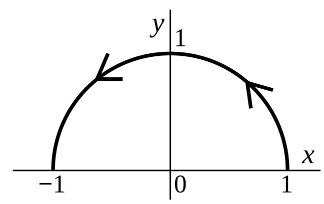
is a gradient vector field. Find a potential for $\mathbf{V}$ and compute the line integral $\int_C \mathbf{V}\cdot\mathbf{T}\,ds$ when C is a curve starting at (0,2) and ending at $(3,1)$.

(16) 4. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x,y,z) = xy^2 + z^4$ for points (x,y,z) in $\mathbb{R}^3$ satisfying $x^2+y^2+z^2=1$.

(12) 5. Find the volume of the finite solid which is between the paraboloids $z = x^2+y^2-1$ and $z = 5-2x^2-2y^2$.

(15) 6. Compute $\int_C y\sin(x^3)\,dx + x\,dy$ where C is the upper half of the unit circle, oriented counterclockwise.

Hint Do not attempt to compute this directly. Instead, use Green's Theorem on the upper half of the region inside the unit circle, and also compute a horizontal line integral. Both the area integral and the "other" line integral should be easy. These results then can be used to solve the problem.



(16) 7. Integrate the function y over the region in the first quadrant of the xy-plane bounded by the curves $xy=1$, $xy=2$, $x^2y=3$ and $x^2y=4$ **using the change of variables technique**. I suggest you try the variables $s=xy$ and $t=x^2y$. Describe the corresponding region in the st-plane, compute the area distortion factor (Jacobian), and rewrite the double integral as a $ds\,dt$ integral.

Comment The function, the Jacobian, and the region *should* interact to produce a result easy to deal with, since this is an invented example. The answer is $\frac{7}{36}$.

Second Exam for Math 291, section 1

April 21, 2003

NAME ____

Do all problems, in any order.
No notes or texts may be used on this exam.
You may use a calculator during the last 20 minutes of the exam.

Problem Number	Possible Points	Points Earned:
1	15	
2	14	
3	12	
4	16	
5	12	
6	15	
7	16	
Total Points Earned:		