# Formula sheet for the second exam in Math 291, fall 2002

FIRST VERSION 11/25/2002; REVISED 11/26/2002

### Lagrange multipliers for one constraint

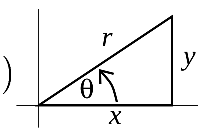
If $G$(the variables) = a constant is the constraint and we want to extremize the objective function, $F$(the variables), then the extreme values can be found among $F$'s values of the solutions of the system of equations $\nabla G = \lambda \nabla F$ (a vector abbreviation for the equations $\lambda \frac{\partial F}{\partial \star} = \frac{\partial G}{\partial \star}$ where $\star$ is each of the variables) **and** the constraint equation.

### Polar coordinates

$x = r\cos\theta \quad y = r\sin\theta$

$r^2 = x^2 + y^2 \quad \theta = \arctan\left(\frac{y}{x}\right)$

$dA = r\,dr\,d\theta$

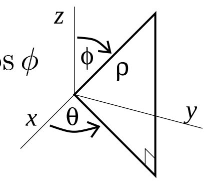



### Spherical coordinates

$x = \rho\sin\phi\cos\theta \quad y = \rho\sin\phi\sin\theta \quad z = \rho\cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

$dV = \rho^2 \sin\phi \, d\rho\, d\theta\, d\phi$




### Change of variables in 2 dimensions

$\int\int_R f(x,y)\,dA = \int\int_{\tilde{R}} f(x(u,v),y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\,dv$; $\frac{\partial(x,y)}{\partial(u,v)} = \det\begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{pmatrix}$, the Jacobian.

**Total mass** of a mass distribution $\rho(x,y,z)$ over a region $R$ of $\mathbb{R}^3$ is $\int\int\int_R \rho(x,y,z)\,dV$.

### Line integral formulas

$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$\int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot \mathbf{r}'(t)\,dt = \int_C \mathbf{F}\cdot\mathbf{T}\,ds$

$\int_C P(x,y)\,dx + Q(x,y)\,dy = \int_a^b P(x(t),y(t))x'(t)\,dt + Q(x(t),y(t))y'(t)\,dt$

### Green's Theorem

$\int_C P\,dx + Q\,dy = \int\int_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$

These $P, Q$ pairs will give $R$'s area $\begin{cases} P=-y \text{ and } Q=0 \\ P=0 \text{ and } Q=x \\ P=-\frac{1}{2}y \text{ and } Q=\frac{1}{2}x \end{cases}$

A **conservative vector field** $\mathbf{V} = P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ is a **gradient vector field**: there's $f(x,y)$ with $\nabla f = \mathbf{V}$ so $\frac{\partial f}{\partial x} = P$ and $\frac{\partial f}{\partial y} = Q$. $f$ is a **potential** for $\mathbf{V}$. A conservative vector field is **path independent**. Work done by such a vector field over a **closed curve** is 0. For $V$ conservative with potential $f$: $\int_C P\,dx + Q\,dy = f(\text{THE END}) - f(\text{THE START})$.

If $P(x,y)\mathbf{i} + Q(x,y)\mathbf{j}$ is conservative, then $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$. If the region is **simply connected** (means **no holes**) then the converse is true, and $f$ is both $\int P(x,y)\,dx$ and $\int Q(x,y)\,dy$.