Useful information for the first exam in Math 291:01, Spring 2003

The first exam will be given in **Hill 425** on **Thursday, March 13, at 4:30 PM** (the usual time but another place). The exam will cover what we have done up to now, including material up to and including section 14.7 of the text. Note that some of the emphasis of the class meetings has been different from what's in the text. I have reserved the room above for periods 6 and 7. Some rules for the exam:

• No books or notes.

There will be a formula sheet you can use on the exam. A draft will be on the web. Please send me comments about it on or before Wednesday, March 12. You'll get copies of the sheet with the exam.

• You may use a calculator only during the last 20 minutes of the exam.

Please leave answers in "unsimplified" form – so $15^2+(.07)\cdot(93.7)$ is preferred to 231.559. You should know simple exact values of transcendental functions such as $\cos\left(\frac{\pi}{2}\right)$ and $\exp(0)$. Traditional math constants such as π and e should be left "as is" and not approximated.

The fall 291 first exam (with answers!) is available on the web, as is the fall 291 review sheet for the first exam (with most answers). There are some differences in emphasis and content between the fall 291 course and this course. The exam will reflect these.

I'll have a **review session** on **Tuesday, March 11, in Hill 525 at 7:30 PM** to discuss these problems and other questions. I hope students will send me *plain text* e-mail containing solutions to the problems with their names. I will proofread these messages and put them on the web. There are more problems below than will appear on the exam.

1. BERGKNOFF Suppose that $F(s,t)$ is a differentiable function of two variables and that $F(5,3)=A$, $\frac{\partial F}{\partial s}(5,3)=B$, $\frac{\partial F}{\partial t}(5,3)=C$, $\frac{\partial^2 F}{\partial s^2}(5,3)=D$, $\frac{\partial^2 F}{\partial s \partial t}(5,3)=E$, and $\frac{\partial^2 F}{\partial t^2}(5,3)=F$. If $G(x,y)=F(x^2+y^2,x^2-y^2)$, compute $\frac{\partial^2 G}{\partial x^2}(2,1)$ in terms of the information given.

2. BERKOWITZ Find an equation for the plane tangent to $z=x^2e^y$ at the point where $x=3$ and $y=0$. Find parametric equations for a line normal to this plane at that point.

3. BROWN The surfaces $x^3+5y^2z-4z^2=3$ and $y^4-7z-2x^2=5$ intersect in a curve, C. The point $p=(3,2,-1)$ is on C. Find parametric equations for a line tangent to C at p.

4. CHOU A particle moves so its position at time t is given by $\mathbf{r}(t)=\cos(t^2)\mathbf{i}+\sin(t^2)\mathbf{j}+t\mathbf{k}$. Describe the geometric path this particle takes as well as you can (you may include a sketch). Based on your conception of the path, predict the asymptotic behavior of $\kappa(t)$, the curvature, and $\tau(t)$, the torsion, as $t\to+\infty$.

Comment You may check algebraically your prediction of the asymptotic behavior of either or both of κ and τ if you wish.

5. ELKHOLY Consider the function $f(x,y,z)=\begin{cases} x-2y+5z, & \text{if } (x,y,z)\neq(0,0,0) \\ 10, & \text{if } (x,y,z)=(0,0,0) \end{cases}$.

a) Use the definition of continuity to verify that f is continuous if $(x,y,z)=(2,6,-7)$.

b) Use the definition of continuity to verify that f is not continuous if $(x,y,z)=(0,0,0)$.

6. FELTZ Suppose $f(x,y)=\begin{cases} 1, & \text{if } x^2+y^2<1 \\ 0, & \text{otherwise} \end{cases}$.

a) Use the definition of continuity to verify that f is not continuous at $(1,0)$.

b) Use the definition of continuity to verify that f is continuous at $(0,0)$.

7. HAWKINS a) If $F(x,y)=\frac{x^2y}{x^2+y^2}$ for $(x,y)\neq(0,0)$, does $\lim\limits_{(x,y)\to(0,0)} F(x,y)$ exist? Give evidence to support your answer.

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b) If $G(x, y) = \frac{x^2y^2}{x^4+y^4}$ for $(x, y) \neq (0, 0)$, does $\lim_{(x,y)\to(0,0)} G(x, y)$ exist? Give evidence to support your answer.

8. HEDDY a) What is the sum, $\mathbf{V}$, of the twelve vectors from the center of a clock to the hours?

b) If the 4 o'clock vector is removed, find $\mathbf{V}$ for the other eleven vectors.

c) If the vectors to 1, 2, 3 are cut in half, find $\mathbf{V}$ for the twelve vectors.

Source *Calculus* by Gilbert Strang.

9. KLEYZIT Suppose S is the surface defined by the equation $z\cos(x^3+y^2)+xz^2+2 = 0$. The point $(-1, 1, 2)$ is on S. Assume that the equation defines x implicitly as a differentiable function of y and z near $(-1, 1, 2)$. If y is changed from 1 to 1.03 and z is changed from 2 to 1.96, use linear approximation to find an approximate x so that $(x, 1.03, 1.96)$ is on S.

Note You are *not* asked to solve a nonlinear equation. You are asked to use a certain approximation strategy to get an answer.

10. MCGOWAN Find the area of the triangle whose vertices are the points $A = (-1, 2, 1)$ and $B = (2, -1, 0)$ and $C = (0, 0, 4)$ in $\mathbb{R}^3$. Also find the coordinates of a point D which together with A and B and C will form the vertices of a parallelogram.

11. PANASENKO The position vector of a point is $\mathbf{r}(t) = e^t\cos(2t)\mathbf{i} + e^{3t}\sin t\mathbf{j} + 4t\mathbf{k}$.

a) Write an integral whose value is the length of the path the particle travels from $t = 0$ to $t = 2\pi$. Do *not* evaluate the integral.

b) Write the acceleration vector of the particle when $t = 0$ as a sum of vectors which are tangent and normal to the path of the particle when $t = 0$.

12. PASHKOVA Find the point (x, y) in the plane for which the sum of the squares of the distances from the three points (a_i, b_i), $i = 1, 2, 3$, is a minimum. Give a physical interpretation of your answer. **Source** An MIT calculus exam.

13. TOZOUR Find and classify all the critical points of these functions.

a) $K(x, y) = 3x^2 + 6xy + 12y^3 + 12x - 24y$. **Source** An MIT calculus exam.

b) $J(x, y) = 8xy - x^4 - y^4$.

14. WAXMAN Sketch the three level curves of the function $W(x, y) = ye^x$ which pass through the points $P = (0, 2)$ and $Q = (2, 0)$ and $R = (1, -1)$. **Label each curve** with the **appropriate function value**. Be sure that your drawing is clear and unambiguous. Also, sketch on the same axes the gradient vector ∇W at the points P and Q and R and S and T. The point S is $(0, -2)$ and the point T is $(-2, 0)$.

Comment This problem was on the Math 291 final exam last semester. People did not do as well on it as I expected.

15. WILSON Sketch $z = x^2 + y$ in $\mathbb{R}^3$. The normal line to the surface at a point is the line perpendicular to the tangent plane to the surface at that point. Find all normal lines to $z = x^2 + y$ which pass through $(0, 0, 3)$.

16. WU Suppose $G(x, y, z) = z^2e^{5x-y^2}$.

a) What is ∇G at the point $p = (1, 2, 3)$?

b) What is the direction of greatest increase of G at p, and what is the directional derivative of G in that direction?

c) Find one direction in which the directional derivative of G at p is 0.

d) What is $D_{\mathbf{u}}G(p)$ if $\mathbf{u}$ is the unit vector in the direction from $(1, 1, 0)$ to $(0, 2, 1)$?