Useful information for the second exam in Math 291, Spring 2003

The time, date, and place of the exam will be:

Hill Center 423, Monday, April 21, at 4:30 PM

This exam will principally cover material presented since the first exam: sections **14.8, 15.1-15.4, 15.7-15.8, 12.7, 16.1-16.4** of the text. Some of the emphasis of the class meetings has been different from what's in the text. I have reserved the room above for periods 6 and 7.

The rules for this exam, as for the first:

- No books or notes.

There will be a formula sheet you can use. A draft will be on the web. Please send me comments about it before Monday, April 21. You'll get copies of the sheet with the exam.

- You may use a calculator only during the last 20 minutes of the exam.

Please leave answers in "unsimplified" form – so $15^2+(.07)\cdot(93.7)$ is preferred to 231.559. You should know simple exact values of transcendental functions such as $\cos\left(\frac{\pi}{2}\right)$ and $\exp(0)$. Traditional math constants such as π and e should be left "as is" and not approximated.

- Show your work: an answer alone may not receive full credit.

Below is a list of problems I gave the students in the fall 291 course to work on as review at a similar time in that course. **Solutions** to most of these problems are linked to that course web page. Also available there is a **real exam** together with answers. I will be available for questions on **Sunday, April 20, at 3:00 PM**. I'll be in my office (Hill 542). If several students come, we will move across the hall to Hill 525.

1. Use Lagrange multipliers to find the maximum and minimum values of $f(x,y)=x+y^2$ subject to the constraint $x^2+2y^2=4$.

2. Evaluate the iterated integral $\int_0^1\int_0^v \sqrt{1-v^2}\,du\,dv$.

3. Change the integral $\int_0^a\int_0^{\sqrt{a^2-y^2}} (x^2+y^2)^{3/2}\,dx\,dy$ to polar coordinates.

4. Evaluate the integral $\int_0^3\int_{y^2}^9 y\cos(x^2)\,dx\,dy$ by reversing the order of integration.

5. Set up a double integral in polar coordinates to find the volume of the solid that lies under the paraboloid $z=x^2+y^2$, above the xy-plane, and inside the cylinder $x^2+y^2=2x$. **Do not evaluate the integral**.

6. Set up a triple integral in spherical coordinates to find $\int\int\int_E x^2\,dV$, where E lies between the spheres $\rho=1$ and $\rho=3$ and above the cone $\phi=\pi/4$. **Do not evaluate the integral**.

7. Evaluate $\int_C (x-2y^2)\,dy$ where C is the arc of the parabola $y=x^2$ from $(-2,4)$ to $(1,1)$.

8. (Lagrange multipliers) Let $f(x,y,z)=xy+xz+yz$.
(a) Find the maximum value of f on the sphere $x^2+y^2+z^2=1$.
(b) Find the minimum value of f on the sphere $x^2+y^2+z^2=1$.

9. Evaluate the integral $\int_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F}(x,y)=x^2y^3\mathbf{i}-y\sqrt{x}\mathbf{j}$ and $\mathbf{r}(t)=t^2\mathbf{i}-t^3\mathbf{j}$, $0\le t\le 1$.

10. Evaluate the integral $\int_0^1\int_0^1 e^{\max(x^2,y^2)}\,dy\,dx$ where $\max(x^2,y^2)$ means the larger of the numbers x^2 and y^2.

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11. I) Set up the triple integral $\iiint_R f(x,y,z)\,dV$ where R is the region inside the sphere $x^2+y^2+z^2=2$ and inside the cone $z^2=x^2+y^2$ (inside the cone means $z^2 \geq x^2+y^2$) in the following systems of coordinates: a) Rectangular; b) Cylindrical; c) Spherical.
II) Evaluate the volume of the region R described in part I.

12. The constraint $x^4+x^2y^2+2y^4+z^4=1$ defines a closed and bounded set in $\mathbb{R}^3$, and thus the continuous function $f(x,y,z)=xyz$ attains its maximum value on that set. What is the maximum value of xyz subject to this constraint? Be sure to analyze carefully and completely any system of equations you solve.

13. Evaluate the line integral with respect to arc length: $\int_C xy\,ds$ where C is the upper right quarter of the ellipse $x^2+4y^2=4$ (i.e. the set of points such that $x^2+4y^2=4$ and $x \geq 0, y \geq 0$).

14. Find the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}=2xy\mathbf{i}+x^2\mathbf{j}$ and C is an arc of a circle centered at $(0,1)$, of radius 1 joining the points $P_1(0,0)$ and $P_2(0,2)$.

15. Suppose R is the trapezoid with vertices $(1,0)$, $(2,0)$, $(0,-2)$, and $(0,-1)$. Use the change of variables $u=x+y$ and $v=x-y$ to integrate $\iint_R e^{(x+y)/(x-y)}\,dA$.

16. Use Green's Theorem to evaluate the line integral $\int_C (y^2-\tan^{-1}x)\,dx+(3x+\sin y)\,dy$, where C is the boundary of the region enclosed by $y=x^2$ and $y=4$.

17. Show that $(2xy^2-1)\mathbf{i}+(6y+2x^2y)\mathbf{j}$ is a gradient vector field, and evaluate $\int_\Gamma (2xy^2-1)\,dx+(6y+2x^2y)\,dy$ where Γ is the graph of $y=(\cos x)^{10}$ from $x=0$ to $x=\frac{\pi}{2}$.

18. Use Green's Theorem to evaluate $\int_\Gamma x^2y\,dx-xy^2\,dy$ where Γ is the circle $x^2+y^2=4$.

19. The plane curve C_1 is a rectangle whose corners are $(0,0)$, $(5,0)$, $(5,4)$, and $(0,4)$, oriented in the usual (counterclockwise) fashion. The plane curve C_2 is the circle of radius 1 whose center is $(2,2)$, oriented in the usual (counterclockwise) fashion. Compute $\int_{C_1} M dx+N dy$ if the following information is known:

i) M and N are continuously differentiable functions of x and y in the region *between* the two curves. In that region, $\frac{\partial M}{\partial x}=\arctan(x^3)$, $\frac{\partial M}{\partial y}=3y$, $\frac{\partial N}{\partial x}=5$, and $\frac{\partial N}{\partial y}=\cos(\sqrt{y})$.
ii) $\int_{C_2} M dx+N dy=8$

20. a) Compute $\int_0^1\int_0^x\int_0^{y^2} xy^2z^3\,dz\,dy\,dx$.
b) Write this iterated integral in "$dx\ dy\ dz$" order. You may want to begin by sketching the volume over which the triple integral is evaluated. You are **not** asked to evaluate the "$dx\,dy\,dz$" result.