Useful information concerning the FINAL EXAM in Math 291, Fall 2002

The place, day, and time of the exam will be **Friday, May 9, 4:00-7:00 PM, SEC 212**. The final exam will be cumulative, covering all of the material in the course. There will be slightly more emphasis on material discussed since the last exam. The exam rules will be unchanged. A comprehensive formula sheet will be included with the exam.

I will have office hours before the final, to be announced on the web. Also I'd like to have a review session before the final, if students can agree about a good time. Again, information will appear on the class web page. The following problem will be on the final exam:

Suppose $\mathbf{F}(x,y,z) = P(x,y,z)\,\mathbf{i} + Q(x,y,z)\,\mathbf{j} + R(x,y,z)\,\mathbf{k}$ where P, Q, and R are continuously differentiable functions. Suppose C is the surface of the unit cube in $\mathbb{R}^3$ ($0 \le x \le 1$ and $0 \le y \le 1$ and $0 \le z \le 1$), $\mathbf{n}$ is the outward unit normal to C, and D is the interior of the unit cube. Prove that $\iint_C \mathbf{F}\cdot\mathbf{n}\,dS = \iiint_D \operatorname{div}\mathbf{F}(x,y,z)\,dV$.

Below are problems from past Math 291 exams which ask about material covered since the last exam. **Answers** to these problems will be linked to the course web page. Also useful are previous exams and review sheets in this course, last semester's final exam (linked to the course web page), and exams in a similar course at MIT (linked to the course web page). Remember: If I thought it was important then, I probably still think it is important!

1. Compute the following integrals. Use Green's theorem, Stokes' theorem or the divergence theorem wherever they are helpful.

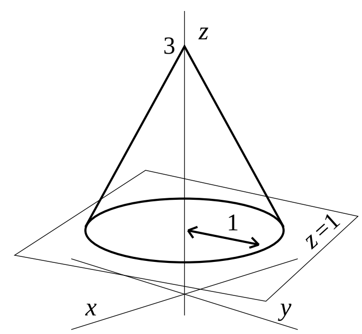
a) $\iint_D xy\,dA$, where D is the triangle in the xy-plane with vertices (0,0), (2,0), and (0,2).

b) $\int_C \mathbf{F}\cdot d\mathbf{s}$, where $\mathbf{F}(x,y) = x^2\mathbf{i} - xy\mathbf{j}$ and C is the segment of the parabola $y = x^2$ beginning at $(-1,1)$ and ending at $(1,1)$.

c) $\iint_S (x^3\mathbf{i} + y^3\mathbf{j} + \cos xy\mathbf{k})\cdot\mathbf{n}\,dS$, where S is the unit sphere and $\mathbf{n}$ points inward.

d) $\iint_S z^2\,dS$, where S is the surface $z = \sqrt{x^2+y^2}$, $0 \le z \le 1$.

e) $\iint_S (\nabla\times\mathbf{F})\cdot\mathbf{n}\,dS$, where $\mathbf{F}(x,y,z) = yz\mathbf{i} + xz^2\mathbf{j} + z^3\mathbf{k}$ and S is the lateral surface of the cone as shown, with $\mathbf{n}$ pointing outward.



Note I didn't get the picture from the instructor who gave me this problem, so I drew what I hope is a suitable picture here. I think the problem can be done with a standard result of vector calculus (and maybe even by a heroic direct computation!).

2. Compute $\int_C e^x \sin z\,dx + y^2\,dy + e^x\cos z\,dz$, where C is the oriented curve $\mathbf{x}(t) = (\cos t)^3\,\mathbf{i} + (\sin t)^3\,\mathbf{j} + t\mathbf{k}$, $0 \le t \le \pi/2$. First find a potential function.

3. A fluid has density 1500 and velocity field $\mathbf{v} = -y\,\mathbf{i} + x\,\mathbf{j} + 2z\,\mathbf{k}$. Find the flow outward through the sphere $x^2+y^2+z^2 = 25$.

4. Sketch the region E contained between the surfaces $z = x^2+y^2$ and $z = 2 - x^2 - y^2$ and let S be the boundary of E.

a) Find the volume of E.

b) Let $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. Find $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ where $\mathbf{n}$ is the outer normal to S.