Please write solutions to two of these problems. Hand them in **Monday, February 10**. *The written solutions should be accompanied by explanations using complete English sentences. You must select* one *of the first pair of problems (#1 and #2) and one of the* one *of the second pair of problems (#3 and #4). Students may work alone or in groups. All students working in a group should contribute to the writeup and should sign what is handed in.*

1. Find equations for two orthogonal planes both of which contain the line $\mathbf{v} = (1, 0, 3) + t(-1, 2, 1)$, one of which passes through the origin.

2. Find the distance between the pair of skew lines given below ("skew" in this case means a bit more than non-intersecting – it means that they are non-intersecting *and* not parallel):

$$\text{The line } L_1 \text{ is } \begin{cases} x = 2t + 1 \\ y = -t - 1 \\ z = 3t \end{cases}. \quad \text{The line } L_2 \text{ is } \begin{cases} x = 3s + 2 \\ y = 5s - 2 \\ z = -4 \end{cases}.$$

Comments: I used different letters (t and s) for the parameters in the two lines. This was to help understanding, because these letters are "dummy" variables (similar logically to the letters in a definite integral: surely $\int_0^1 w^2 dw$ and $\int_0^1 u^2 du$ are the same).

3. Is the point (1,2,3) on a tangent line of the twisted cubic $\mathbf{c}(t) = (t, t^2, t^3)$?

4. A function $c : \mathbb{R} \to \mathbb{R}^2$ is called *smooth* if c is differentiable. Physically such functions should represent motion that has no jerks or kinks.

a) Suppose $q : \mathbb{R} \to \mathbb{R}$ is defined by $q(x) = \begin{cases} x^{100} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$. Sketch $y = q(x)$ and prove that q is differentiable. (Do this by first computing the derivative algorithmically for $x < 0$ and $x > 0$ – this is easy. Compute $q'(0)$ by looking directly at the definition and computing the limit from both sides.)

b) Sketch the smooth curve defined by $c(t) = (q(t), q(-t))$.

c) Explain why you've drawn a picture of smooth motion. (Hardest part of the question!)