Please write solutions to two of these problems. Hand them in Wednesday, February 26. The written solutions should be accompanied by explanations using complete English sentences. Students may work alone or in pairs. *Both students working together should contribute to the writeup and* proofread *all of it.*

1. a) Compute the distance (which we will call $\mathcal{D}(A,B,C,D)$) from the origin, $(0,0,0)$, to the plane $Ax+By+Cz=D$.

b) Describe the *domain* of the function $\mathcal{D}$. It will be a collection of points in $\mathbb{R}^4$, each of which corresponds to a "real" plane in $\mathbb{R}^3$. Not every point in $\mathbb{R}^4$ corresponds to a "real" plane in $\mathbb{R}^3$! What is the *range* of $\mathcal{D}$ (the collection of all possible values)?

c) If λ is any non-zero real number, then the function $\mathcal{D}$ has the following property:

$$\mathcal{D}(\lambda A,\lambda B,\lambda C,\lambda D)=\mathcal{D}(A,B,C,D)\,.$$

Prove this equation algebraically from your formula for $\mathcal{D}$. Explain why it is true geometrically after you do the computation.

d) The graph of $\mathcal{D}$ is a subset of $\mathbb{R}^5$. What does the graph of the section of the graph obtained by constraining B and C to be 0 look like? (This will be a subset of $\mathbb{R}^3$.) What does the graph of the section obtained by constraining C and D to be 0 look like? Are your graphs reasonable?

2. a) The function $A\colon\mathbb{R}^3\to\mathbb{R}$ is defined by $A(x,y,z)=\max(|x|,|y|,|z|)$. Sketch the level set or contour surface L_A consisting of (x,y,z)'s with $A(x,y,z)=1$.

b) The function $B\colon\mathbb{R}^3\to\mathbb{R}$ is defined by $B(x,y,z)=|x|+|y|+|z|$. Sketch the level set or contour surface L_B consisting of (x,y,z)'s with $B(x,y,z)=1$.

c) One of the sets (L_A, L_B) is inside the other. Decide which is inside, and verify your statement *algebraically*, without using your sketches.

Definitions of "inside" for this problem
A point (x,y,z) is inside L_A if $A(x,y,z)\le 1$. A point (x,y,z) is inside L_B if $B(x,y,z)\le 1$

3. Suppose $h(x,y)=\begin{cases}0, & \text{if } x>0 \text{ \underline{and} } y>0\\ 1, & \text{otherwise}\end{cases}$. Then consider the following statement:

> Fix (x_0,y_0) in $\mathbb{R}^2$. If any $K>0$ is given, then there is a positive real number H, which may depend on (x_0,y_0) and on K, so that if $\|(x,y)-(x_0,y_0)\|<H$, then $|h(x,y)-h(x_0,y_0)|<K$.

For which (x_0,y_0) in $\mathbb{R}^2$ is this statement true, and for which is it false? If you assert it is true for a point (x_0,y_0), you must provide an H, and explain why your H works. If you assert it is false, you must give an example of a K and explain why there is no H which will make the statement true.

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4. Suppose $g(x,y) = \begin{cases} 0, & \text{if } y \neq 0 \\ x, & \text{if } y = 0 \end{cases}$. Then consider the following statement:

> Fix (x_0, y_0) in $\mathbb{R}^2$. If any $K > 0$ is given, then there is a positive real number H, which may depend on (x_0, y_0) and on K, so that if $\|(x,y)-(x_0,y_0)\| < H$, then $|g(x,y) - g(x_0,y_0)| < K$.

For which (x_0, y_0) in $\mathbb{R}^2$ is this statement true, and for which is it false? If you assert it is true for a point (x_0, y_0), you must provide an H, and explain why your H works. If you assert it is false, you must give an example of a K and explain why there is <u>no</u> H which will make the statement true.

Comments for problems 3 and 4 I'd probably start each of these problems by drawing a graph since I like pictures. I'd begin to analyze the problem at a few specific (x_0, y_0)'s, such as $(0,0)$ and $(1,0)$ and $(1,1)$, with some specific K's, such as 1 and $\frac{1}{2}$ and $\frac{1}{10{,}000}$, and go on to generalize my reasoning. This is only to help you begin this problem. Your writeup should give the general solution.

5. A rectangular box with an open top has a square base. The sides are made of cardboard, costing 3 cents per square foot. The base is made of plywood, costing 50 cents per square foot. The box should have a capacity of no more than 10 cubic feet and no less than 2 cubic feet. At the same time, due to limitations of construction, no edge of the box should be shorter than 3 inches or longer than 36 inches. Find a plausible domain for the dimensions of the box based on these specifications and describe the domain carefully, algebraically. Sketch the domain in $\mathbb{R}^2$. (You *must* give a complete algebraic description of the domain, however. The picture is *not* a substitute for this description.) Write a formula for a function which calculates the cost of the materials in each possible box.