#6 **Problems for 291:01** 2/26/2003

*Please write solutions to two of these problems. Hand them in Wednesday, March 5. The written solutions should be accompanied by explanations using complete English sentences. Students should work* **alone**. *They may ask me questions.*

1. a) Suppose $h(x,y) = x^y$. Rewrite $h$ as a composition of standard functions, and then find the domain of $h$ and the first partial derivatives of $h$ based on the standard restrictions. I know that $2^3 = 8$. If $x$ is increased by .01 (so $(x,y)$ changes from $(2,3)$ to $(2.01,3)$), approximate the change in $h$ using the linear approximation. If $y$ is increased by .01 (so $(x,y)$ changes from $(2,3)$ to $(2,3.01)$), approximate the change in $h$ using the linear approximation. Compare the "exact answers" to the linearization answers.

b) Suppose $j(x,y,z) = x^{(y^z)}$. Rewrite $j$ as a composition of standard functions, and then find the domain of $j$ and the first partial derivatives of $j$ based on the standard restrictions. I know that $2^{(3^4)} \approx 2.417 \cdot 10^{24}$*. If one of the variables is increased by .01, which variable will likely make the biggest change in the value of $j$? Support your assertion by an argument using linear approximation based on the derivatives which have been calculated**.

2. a) Suppose $f(u,v)$ is a differentiable function of $u$ and $v$, and that $u = e^x \cos y$ and $v = e^x \sin y$. Express $\frac{\partial f}{\partial x}$ in terms of $\frac{\partial f}{\partial u}$, $\frac{\partial f}{\partial v}$, and functions involving $x$ and $y$.

b) Note that if $x = 0$ and $y = \frac{\pi}{2}$ then $u = 0$ and $v = 1$. Suppose also that you know $\frac{\partial f}{\partial u}(0,1) = 7$ and $\frac{\partial f}{\partial v}(0,1) = -3$. Use this information together with your formula in a) to compute $\frac{\partial f}{\partial x}$ when $x = 0$ and $y = \frac{\pi}{2}$.

3. Suppose $R(x,y) = v(x+y^2)$ where $v$ is a four times differentiable function of *one* variable. Suppose you know also that:

$$v(0) = \alpha,\ v'(0) = \beta,\ v^{(2)}(0) = \gamma,\ v^{(3)}(0) = \delta,\ v^{(4)}(0) = \epsilon$$

Compute the seven quantities:

$$R(0,0),\ \frac{\partial R}{\partial x}(0,0),\ \frac{\partial R}{\partial y}(0,0),\ \frac{\partial^2 R}{\partial x^2}(0,0),\ \frac{\partial^2 R}{\partial y^2}(0,0),\ \frac{\partial^2 R}{\partial x \partial y}(0,0),\ \frac{\partial^4 R}{\partial x^2 \partial y^2}(0,0)$$

in terms of $\alpha$, $\beta$, $\gamma$, $\delta$, and $\epsilon$.

4. Suppose $g(t) = Q(t^3, t^5)$. Suppose you also know that

$$Q(1,1) = A, \frac{\partial Q}{\partial x}(1,1) = B, \frac{\partial Q}{\partial y}(1,1) = C, \frac{\partial^2 Q}{\partial x^2}(1,1) = D, \frac{\partial^2 Q}{\partial x \partial y}(1,1) = E, \frac{\partial^2 Q}{\partial y^2}(1,1) = F$$

Compute the quantities $g(1)$, $g'(1)$, and $g''(1)$ in terms of $A$, $B$, $C$, $D$, and $E$.

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* Actually, it is *exactly* 24178 51639 22925 83494 12352.

** Please note that I am *not* interested in the exact values of the perturbed function here, only in the linearized approximations to the perturbed function values.