

Please write solutions to two of these problems. Hand them in Thursday, April 10. The written solutions should be accompanied by explanations using complete English sentences. Students should work **alone**. They may ask me questions.

1. Consider the tetrahedron T whose corners are $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$. Suppose the function f is defined by $f(x, y, z) = x^3 + y^2 + z$. The goal of this problem is to use **Maple** to compute $\iiint_T f \, dV$

a) Write the integral as an iterated integral in the order $dx \, dy \, dz$. Have **Maple** do each of the integrals, and present a printout of the computations.

b) Write the integral as an iterated integral in the order $dy \, dz \, dx$. Have **Maple** do each of the integrals, and present a printout of the computations.

c) Write the integral as an iterated integral in the order $dz \, dx \, dy$. Have **Maple** do each of the integrals, and present a printout of the computations.

d) (Optional) Write an essay of at most 100 words describing the irritation you'd have computing this yourself.

2. a) Verify the equation (that is, do the computation)

$$\int_0^1 \int_0^{1-x} \exp\left(\frac{y}{x+y}\right) dy \, dx = \frac{e-1}{2}$$

by using the transformation $\begin{cases} x+y = u \\ y = uv \end{cases}$ *

b) Use a linear change of variables to find the area inside the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (Try $au = x$ and $bv = y$. Please show how the change of variables formula works in this case.)

3. Suppose \mathcal{S} is the unit square where $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Does

$$\iint_{\mathcal{S}} \frac{1}{x+y} \, dA$$

converge, and, if it does, what is its value? Include a simple sketch produced by **Maple** of the volume indicated in this integral.

4. Compute:

$$\text{a) } \int_0^{\frac{\pi}{2}} \left(\int_y^{\frac{\pi}{2}} \frac{\sin x}{x} \, dx \right) dy \quad \text{b) } \int_0^1 \left(\int_{\sqrt{y}}^1 e^{(7x^3)} \, dx \right) dy \quad \text{c) } \int_0^1 \left(\int_x^{x^{1/3}} \sqrt{1-y^4} \, dy \right) dx$$

5. Does either of these converge? If not, explain why. If yes, evaluate. Of course, explain your answers.

$$\text{a) } \iint_{\mathbb{R}^2} e^{-|x|-|y|} \, dA \quad \text{b) } \iint_{\mathbb{R}^2} e^{-|x+y|} \, dA$$

* This problem is from the Schaum's outline series problem book on advanced calculus.