Math 311, section 1

Entrance "exam"

Due at the beginning of class, Monday, January 27, 2003

1. (12 points) Let $a_1, a_2, a_3, \ldots$ be an infinite sequence of real numbers satisfying: $a_1 = 4$ and $a_2 = 5$ and for all integers $n \geq 3$, $a_n = \dfrac{5a_{n-1}}{6} + \dfrac{3}{a_{n-2}}$. Prove that for every natural number n, $3 \leq a_n \leq 6$.

2. (8 points) In the following sentence, assume that the letter f was previously introduced to represent some function from $\mathbb{R}$ to $\mathbb{R}$ and that the letter a was introduced to represent some number.

> There exists a positive real number u such that for every real number x if $|x - a| < u$ then $f(x) \geq f(a)$.

Construct an English sentence that is equivalent to the negation of the above sentence. The sentence you construct should not contain any words of negation such as "no", "not" or "false". Your sentence may use mathematical notation from arithmetic such as "+","−","×","≤", '≥","≠", etc.

3. (8 points) Prove or disprove the following statement:

> For any three subsets A, B, C of $\mathbb{R}$, if $A \cap B \neq \emptyset$ and $B \cap C \neq \emptyset$ and $A \cap C \neq \emptyset$, then $A \cap B \cap C \neq \emptyset$.

4. (18 points) Determine whether each of the following sentences is true or false and prove your answers.

a) For all positive real numbers x there exists a positive real number y such that $xy < \frac{1}{100}$.
b) There exists a positive real number y such that for all positive real numbers x, $xy < \frac{1}{100}$.
c) There exists a real number y such that for all positive real numbers x, $xy < 0$.

Rules Please treat this as any other homework assignment. That is, you may consult textbooks or acquaintances or me (!), but the written work you hand in must be your own. I will grade what you hand in as an exam. A passing grade will be at least 75% of the 46 points. Familiarity with all of the material tested here is necessary for success in this course.