(14) 1. Suppose the sequence (x_n) is defined by $x_n := \dfrac{n-1}{5n+7}$. Find x so that (x_n) converges to x. Prove your assertion using the definition of convergence.

(12) 2. Suppose that the sequence (x_n) converges to x and the sequence (y_n) converges to y. Prove that the sequence (z_n) defined by $z_n := x_n + y_n$ converges to $x + y$.

(14) 3. Suppose S is a nonempty subset of $\mathbb{R}$ which is bounded above, and $a \in \mathbb{R}$. Define a subset T of $\mathbb{R}$ by $T := \{x : \exists s \in S \text{ so that } x = a + s\}$. ($T$ is S "translated by a".) Prove that T is bounded above, and prove that $\sup T = a + \sup S$.

(12) 4. Prove that $2n^2 < 3^n$ for all $n \in \mathbb{N}$.

Comment You may need to verify more than one example numerically.

(14) 5. Suppose that S is a nonempty subset of $\mathbb{R}$ with the property that if $a \in S$ then $a^2 \in S$. Prove that if S is bounded above, then $\sup S \le 1$.

(14) 6. Suppose that (x_n) is a convergent sequence and (y_n) is such that for any $\varepsilon > 0$ there exists $M(\varepsilon) \in \mathbb{N}$ such that $|x_n - y_n| < \varepsilon$ for all $n \ge M(\varepsilon)$. Does it follow that (y_n) is convergent? Prove your assertion.

Answers to Part 1 of Exam 1

1. **The definition of supremum**
Suppose S is a nonempty subset of $\mathbb{R}$. x is a supremum of S if
a) x is an upper bound of S: for $s \in S$, $s \le x$.
b) If y is an upper bound of S, then $x \le y$.

2. **A criterion for an upper bound to be a supremum**
Suppose x is an upper bound of a nonempty subset of $\mathbb{R}$. x is a supremum of S if and only if for all $\varepsilon > 0$, there is $s \in S$ with $x - \varepsilon < s \le x$.

3. **The Completeness Axiom**
If S is a nonempty subset of R which is bounded above, then S has a least upper bound.

4. **The Archimedean Property**
If $x \in \mathbb{R}$ then there is $n \in \mathbb{N}$ so that $n > x$.

5. **The definition of convergence of a sequence**
A sequence (x_n) converges to $x \in \mathbb{R}$ if for every $\varepsilon > 0$ there is $K(\varepsilon) \in \mathbb{N}$ such that for $n \in \mathbb{N}$ with $n \ge K(\varepsilon)$, $|x_n - x| < \varepsilon$.

Part 2 of the First Exam for Math 311, section 1

March 6, 2003

NAME ____

Do all problems, in any order.
No notes or texts may be used on this exam.
The last page contains the answers to Part 1.

Problem Number	Possible Points	Points Earned:
1	14	
2	12	
3	14	
4	12	
5	14	
6	14	
Total Points Earned:		