311:01 # Part 1 of the Final Exam **5/13/2003**

NAME ____

A) Suppose S is a nonempty subset of $\mathbb{R}$. Define "x is $\sup S$." (Please include an explanation of what "upper bound" means.)

B) Suppose x is an upper bound for a nonempty subset, S of $\mathbb{R}$. State a necessary and sufficient criterion for x to be $\sup S$. (This criterion should not repeat the definition.)

C) State the Completeness Axiom.

D) State the Archimedean Property.

E) Suppose (x_n) is a sequence. Define "(x_n) converges to x."

F) Suppose $f\colon \mathbb{R} \to \mathbb{R}$ and $c \in \mathbb{R}$. Define "f is continuous at $c \in \mathbb{R}$."

G) Suppose $f\colon \mathbb{R} \to \mathbb{R}$ and $c \in \mathbb{R}$. State a sequential criterion which is equivalent to "f is continuous at $c \in \mathbb{R}$."

H) Suppose $\mathcal{P} = \{a = x_0 < x_1 < \ldots < x_{n-1} < x_n = b\}$ is a partition of the closed and bounded interval $[a, b]$, and f is a bounded function defined on $[a, b]$. Define $US(f, \mathcal{P})$, the upper sum for f associated with the partition $\mathcal{P}$ on $[a, b]$.

I) Suppose f is a bounded function defined on the closed and bounded interval $[a, b]$. State a necessary and sufficient criterion directly involving upper and lower sums of f which is equivalent to "f is Riemann integrable in $[a, b]$."

J) Suppose f is Riemann integrable in the closed and bounded interval $[a, b]$. State a version of the Fundamental Theorem of Calculus for f.