Useful information for the first exam in Math 311:01, Spring 2003

The time, date, and place will be:

SEC 212, Thursday, March 6, at 1:10 PM
(The usual time & place on Thursdays.)

The exam will cover the material of chapters 1 and 2 and sections 3.1 and 3.2 as done by this class. No books or notes may be used on the exam. The exam will have two parts.

Part 1 will be worth 20 points. I will ask you to

- State the Completeness Axiom.
  References: page 37 of the text or the course diary entry for 2/7/2003.
- State the Archimedean Property.
  References: page 40 of the text or the course diary entry for 2/7/2003.
- Define $\sup(S)$.
  References: page 25 of the text or the course diary entry for 2/7/2003.
- State a necessary and sufficient criterion for an upper bound to be a supremum.
  References: page 36 of the text or the course diary entry for 2/7/2003.
- Define "The sequence (x_n) converges to x."
  References: page 54 of the text or the course diary entry for 2/20/2003.

As soon as you finish and hand in part 1, I will give you part 2.

Part 2 will contain the answers to the questions of part 1, and will have other questions to answer, similar to those below. The questions below are taken from exams given by four instructors of Math 311. I hope that students will send me *plain text* e-mail containing solutions to the problems with their names. I will proofread these messages and put them on the web. There are many more problems below than will appear on part 2 of the exam.

1. BECKHORN Prove that $2n-3 \le 2^{n-2}$ for all $n \ge 5$, $n \in \mathbb{N}$.

2. BENSON Prove using the definition of limit that $\lim\left(\dfrac{1+2n^2}{3+4n^2}\right) = \dfrac{1}{2}$.

3. CHAN Suppose $a, b \in \mathbb{R}$ and suppose that for every $\varepsilon > 0$ we have $a \le b + \varepsilon$.

a) Prove that $a \le b$.

b) Show by example that it does *not* follow that $a < b$.

4. CHANG If A and B are any nonempty sets of numbers which have the property that $a \le b$ for all $a \in A$ and all $b \in B$, prove:

a) $\sup A \le b$ for all $b \in B$.

b) $\sup A \le \inf B$.

5. CITTERBART If $T := \left\{\dfrac{1}{n} + \dfrac{1}{m} : n, m \in \mathbb{N}\right\}$, find $\inf T$ and $\sup T$. Prove that your answers are correct.

6. COHEN Prove that if $\lim(x_n) = x$ and if $x > 0$, then there is a positive integer M such that $x_n > 0$ for all $n \ge M$.

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7. **GOODE** If $a, b \in \mathbb{R}$ and $b \neq 0$, prove that $\left| \dfrac{a}{b} \right| = \dfrac{|a|}{|b|}$.

8. **GREENBAUM** Suppose (a_n) and (b_n) are any sequences of real numbers with the following properties:

(a_n) converges, and $\lim(a_n)$ is 0.

(b_n) is bounded above by 3 and below -3: so $|b_n| \leq 3$ for all $n \in \mathbb{N}$.

Prove that the sequence (c_n) with c_n defined by $c_n := a_n \cdot b_n$ converges, and that $\lim(c_n) = 0$.

9. **GUTHRIE** Let (x_n), (a_n), (b_n) be infinite sequences such that $a_n = x_{2n}$ and $b_n = x_{2n-1}$ for every $n \in \mathbb{N}$. Let L be a real number. Assume $\lim(a_n) = L$ and $\lim(b_n) = L$. Prove that (x_n) converges and that its limit is L.

10. **HEDBERG** Let (x_n) be an infinite sequence. Assume $\lim(x_n) = L$. Let M be an element of $\mathbb{N}$ and suppose that the set S is defined by $S = \{x_n : n \geq M\}$. Prove that $\inf S \leq L$.

11. **LACOGNATA** Let S be a bounded nonempty set of positive real numbers. Let T be the set defined by $T = \{\frac{1}{x} : x \in S\}$. Prove that $\inf T = \frac{1}{\sup S}$.

12. **LEDUC** Find all $x \in \mathbb{R}$ such that $|2x + 1| = 5 - |x|$.

13. **NUCCI** For each of the following statements, prove (if true) or give a counterexample (if false).

a) Suppose (x_n) is a convergent sequence, and the limit of the sequence is x. If each x_n is irrational, then x is irrational.

b) Suppose (x_n) is a convergent sequence, and the limit of the sequence is x. If each x_n is non-negative (that is, greater than or equal to 0), then x is non-negative.

c) Suppose (x_n) is a convergent sequence, and the limit of the sequence is x. If x is non-negative, then eventually each x_n is non-negative (that is, there is $N \in \mathbb{N}$ so that x_n is non-negative for $n \geq N$).

14. **OLEYNICK** Let a, b, c, d be real numbers such that $a < b$ and $c < d$. Prove the inequality $bd + ac > ad + bc$.

15. **SUNG** Suppose the sequence (x_n) is defined by $x_n := \frac{n}{2n+1}$. Find x so that (x_n) converges to x and prove your assertion.

16. **TROPEANO** Suppose that S is a nonempty bounded subset of $\mathbb{R}$, and $T = \{-2x : x \in S\}$. Why does $\inf T$ exist? Why is $\inf T = -2 \sup S$?

17. **YE** Suppose that U is a nonempty subset of $\mathbb{R}$, that 19 is an upper bound for U, and that $W = \{-2x : x \in U\}$. Does it follow that W has an upper bound? If so, prove it. If not, give an example that shows why not.