Useful information for the final exam in Math 311:01, Spring 2003

The time, date, and place will be:

SEC 205, Tuesday, May 13, from 12:00 to 3:00 PM

I reserved a room for a review session. The time, date, and place are:

Hill 525, Saturday, May 10, at 1 PM

I can review textbook problems, workshop problems, and try to give intellectual outlines (!) of sections of the course.

I will also be in my office (Hill 542) on Monday, May 12, from 1 to 5 PM. Other office hours can be arranged (use e-mail for this, please) and I will also try to answer e-mail inquiries in a timely manner.

The exam will be cumulative over the whole course, with perhaps a slight overemphasis on the material covered since the last exam. We have covered material from chapters 1, 2, 3, 4, 5, and a bit of 6, and have given a treatment of the first three sections of chapter 7 using an approach different from what is in the text. Reference material includes your notes, my diary entries on the web, the text, and Professor Cohen's notes on the Riemann integral (also available on the web).

No books or notes may be used on the exam. The exam will have two parts.

Part 1 will be worth 40 points out of 200. I will ask the following questions.

- Suppose $\mathcal{P} = \{a = x_0 < x_1 < \ldots < x_{n-1} < x_n = b\}$ is a partition of the closed and bounded interval $[a, b]$, and f is a bounded function defined on $[a, b]$. Define the upper $US(f, \mathcal{P})$, the upper sum for f associated with the partition $\mathcal{P}$ on $[a, b]$.

  References: Professor Cohen's notes or the course diary entry for 4/21/2003.
- Suppose f is a bounded function defined on the closed and bounded interval $[a, b]$. State a necessary and sufficient criterion directly involving upper and lower sums of f which is equivalent to "f is Riemann integrable in $[a, b]$."

  References: Professor Cohen's notes or the course diary entry for 4/23/2003.
- Suppose f is Riemann integrable in the closed and bounded interval $[a, b]$. State a version of the Fundamental Theorem of Calculus for f.

  References: Professor Cohen's notes or the course diary entry for 5/5/2003 (I hope – this diary entry isn't written yet!) or 7.3.5 on page 212 of the text.

There will be a total of 10 items to define or state. The balance (7 others) will be selected from questions already given on part 1 of exams 1 and 2.

As soon as you finish and hand in part 1, I will give you part 2.

Part 2 will contain the answers to all the questions of part 1 of exams 1 and 2 and the final exam, and will have other questions to answer, similar in level to those given on exams earlier in the course or on review material given out earlier in the course.

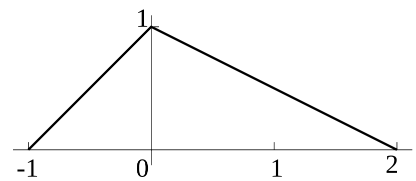
Please consider the problems on Riemann integrals (handed out in class on 4/23/2003 and available from the course home page on the web) as review problems for the last segment of the course. A few additional sample problems follow.

OVER

A. Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \begin{cases} x & \text{if } x \text{ is } \frac{1}{n} \text{ for } n \in \mathbb{N} \\ 0 & \text{for all other } x \in [0,1] \end{cases}$. Is f Riemann integrable on $[0,1]$? If it is, what is $\int_0^1 f$?

B. If f is integrable on $[0,1]$, define $g : [-1,1] \to \mathbb{R}$ by $g(x) = \begin{cases} 0 & \text{if } x < 0 \\ f(x) & \text{if } x \geq 0 \end{cases}$.
Prove that g is integrable on $[-1,1]$ and that $\int_{-1}^1 g = \int_0^1 f$.

C. Suppose f has domain $[-1,2]$ and $f(x) = \begin{cases} x+1 & \text{if } -1 \leq x < 0 \\ 1 - \frac{1}{2}x & \text{if } 0 \leq x \leq 2 \end{cases}$. A picture of the graph of f is kindly included here.



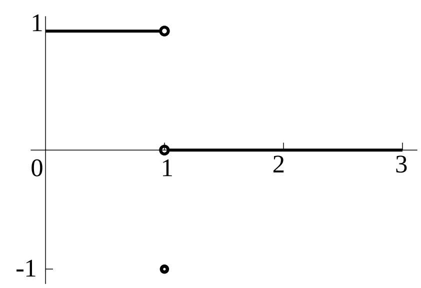
a) Suppose $\mathcal{P}$ is the partition $\{-1, -\frac{1}{2}, 0, 1, \frac{3}{2}, 2\}$ of $[-1,2]$. Compute $US(f,\mathcal{P})$ and $LS(f,\mathcal{P})$.

b) Find a partition $\mathcal{P}$ of $[-1,2]$ so that the difference of $US(f,\mathcal{P})$ and $LS(f,\mathcal{P})$ is less than $\frac{1}{100}$.

c) Explain briefly why f is Riemann integrable on $[-1,2]$.

d) If $F(x) = \int_{-1}^x f$, for which $x \in [-1,2]$ is F continuous? For which $x \in [-1,2]$ is F differentiable? What is $F'(x)$ for those x's?

D. Suppose f has domain $[0,3]$ and $f(x) = \begin{cases} 1 & \text{if } 0 \leq x < 1 \\ -1 & \text{if } x = 1 \\ 0 & \text{if } 1 < x \leq 3 \end{cases}$. A picture of the graph of f is generously included here.



a) Suppose $\mathcal{P}$ is the partition $\{0,1,2,3\}$ of $[0,3]$. Compute $US(f,\mathcal{P})$ and $LS(f,\mathcal{P})$.

b) Find a partition $\mathcal{P}$ of $[0,3]$ so that the difference of $US(f,\mathcal{P})$ and $LS(f,\mathcal{P})$ is less than $\frac{1}{100}$.

c) Explain briefly why f is Riemann integrable on $[0,3]$. What is $\int_0^3 f$?

d) If $F(x) = \int_0^x f$, for which $x \in [0,3]$ is F continuous? For which $x \in [0,3]$ is F differentiable? What is $F'(x)$ for those x's?