#5 **Problems for 311:01** 2/18/2003

Please write solutions to **one** *of these problems and to* **one** *of the problems on the previous workshop list that you have not done before. Hand them in next Thursday, February 27. These written solutions should be accompanied by explanations using complete English sentences. Students may work alone or in groups. All students working in a group should contribute to the writeup and should sign what is handed in.*

1. Let x and a be positive numbers such that $x^3 < a$. Prove that there exists $\varepsilon > 0$ such that $(x+\varepsilon)^3 < a$.

2. Assume that (x_n) is a convergent sequence and $\lim(x_n) = x$. Prove the following:

$$\lim\left(\frac{x_{n+1} + x_{n+2} + x_{n+3} + \cdots + x_{2n}}{n}\right) = x\ .$$

3. (For this problem, assume that we have shown the existence of a positive square root of every positive real number.) Prove directly from the definition of limit that

$$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n}+1} = 1.$$

4. a) Suppose $a < b$, $c < d$, $I = [a, b]$, and $J = [c, d]$. Prove that if $c \in I$ and $d \in I$, then $J \subseteq I$.

b) Suppose S is a bounded nonempty subset of $\mathbb{R}$, and $r = \sup S$ and $t = \inf S$. Prove that $S \subseteq [t, r]$.

c) Suppose S is a bounded nonempty subset of $\mathbb{R}$, and $r = \sup S$ and $t = \inf S$. If $[a, b]$ is a closed bounded interval and $S \subseteq [a, b]$, prove that $[t, r] \subseteq [a, b]$. An "efficient" proof will use part a).