Please write solutions to two of these problems. Hand them in Monday, March 24. The written solutions should be accompanied by explanations using complete English sentences. Students should work **alone**. *They may ask me questions.*

1. Use the Squeeze Theorem to give a simple proof that if $k \in \mathbb{N}$, then $\lim \left(\frac{n^k}{n!}\right)$ exists and is 0.

2. Suppose that $(x_n)_{n\in\mathbb{N}}$ is a sequence with $x_n > 0$ for all n, and that

$$\lim_{n\to\infty} (x_n)^{1/n} = L \text{ exists.}$$

If $L < 1$ prove that

$$\lim_{n\to\infty} x_n = 0$$

Hint Perhaps try this first with some number for L, such as $L = .9$. What happens if you take $\varepsilon = .05$? How big and how small can $(x_n)^{1/n}$ be when n is large enough (depending on ε, of course)? What then will be true about x_n itself? Once you've done this, try to generalize the reasoning to every $L < 1$.

3. Suppose (b_n) is a sequence with the following property:

† For any $n \in \mathbb{N}$ there is an integer m with $m > n$ so that $a_m \cdot a_n < 0$.

a) Find a simple example of a sequence satisfying †.

b) Suppose in addition that (a_n) converges, and its limit is A. What can you say about A? Prove your assertion.

4. Suppose (b_n) is a sequence with the following property:

$\star$ For any $n \in \mathbb{N}$ there is an integer m with $m > n$ so that $b_m > b_n$.

a) Find a simple example of a sequence satisfying $\star$.

b) Are there any examples of non-monotonic sequences satisfying $\star$? (Of course you must read $\star$ very carefully.) If there are, give one example. If there are not, prove your assertion.

c) Suppose in addition that (b_n) converges, and its limit is B. What can you say about the relation of B to any of the individual members of the sequence as a result of $\star$?

5. Let (a_n) be an infinite sequence. Assume:

$$\lim_{n\to\infty} n^4 \left(a_n - 4 - \frac{5}{n^2}\right) = 12.$$

Prove:

$$\lim_{n\to\infty} n^2 (a_n - 4) = 5.$$