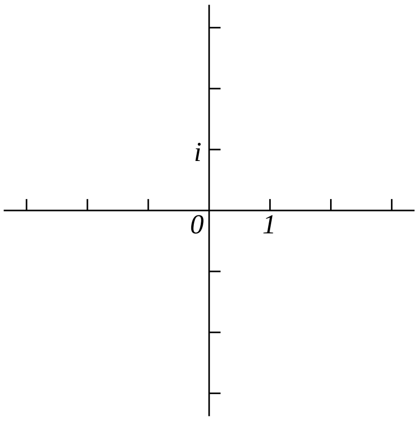
(16) 1. W is the set of complex numbers z satisfying the inequalities $\operatorname{Re} z \geq 0$ and $\operatorname{Im} z \geq 0$ and $|z| \leq 1$. So W is the complex numbers in the first quadrant inside the closed unit circle.

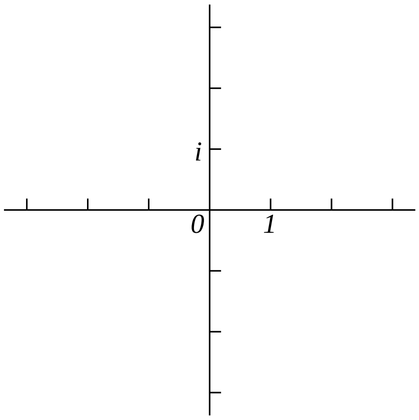
This problem investigates the effect of the mapping $z \to \overline{3iz^2 + 2}$ on W. Complete every part of this problem in order to get full credit.

a) Sketch W on the axes indicated.

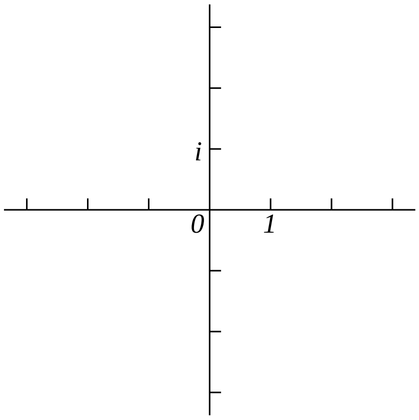


b) Is 0 an interior point of W? **ANSWER:** ________________

c) Suppose W_1 is the image of W under the mapping $z \to z^2$. Sketch W_1 on the axes indicated.

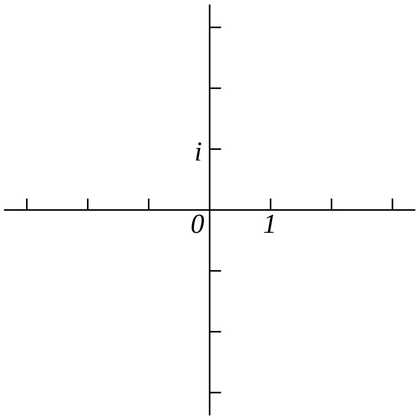


d) Suppose W_2 is the image of W_1 under the mapping $z \to iz$. Sketch W_2 on the axes indicated.

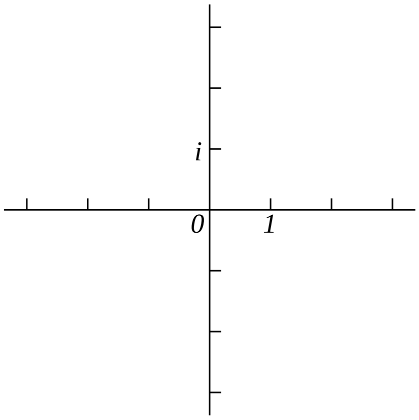


This problem continues on the next page.

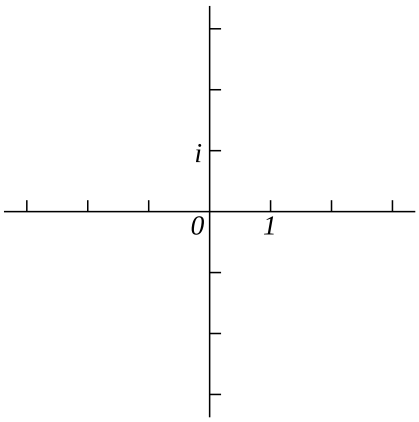
e) Suppose W_3 is the image of W_2 under the mapping $z \to 3z$. Sketch W_3 on the axes indicated.



f) Suppose W_4 is the image of W_3 under the mapping $z \to z + 2$. Sketch W_4 on the axes indicated.



g) Suppose W_5 is the image of W_4 under the mapping $z \to \overline{z}$. Sketch W_5 on the axes indicated.



h) W_5 is the image of W under the mapping $z \to \overline{3iz^2 + 2}$. Is 0 an interior point of W_5?

ANSWER: ________________

(16) 2. a) Find all values of $(-i)^i$.

b) Suppose that b is real and a is any non-zero complex number. Prove that $\left|a^b\right| = \left||a|^b\right|$ for all values of both sides of the equation.

(16) 3. a) Suppose $c(x, y)$ is a harmonic function, so $\dfrac{\partial^2 c}{\partial x^2} + \dfrac{\partial^2 c}{\partial y^2} = 0$.

Prove that $f = \dfrac{\partial c}{\partial x} - i\dfrac{\partial c}{\partial y}$ is complex analytic.

b) Verify that $w(x, y) = \cos x \cosh y$ is harmonic, and find one harmonic conjugate.

(16) 4. a) Compute $\displaystyle\int_C 2x + 3iy^2 \, dz$ where C is the straight line segment from 0 to $1 + i$.

b) If C is the circle of radius 2 centered at 0, verify that

$$\left| \int_C \left(\frac{z+3}{z^3+1} \right) dz \right| \leq \frac{20\pi}{7}.$$

Show details of your estimates.

(16) 5. In all parts of this problem, C is the unit circle oriented in the standard counterclockwise manner. Evaluate the integrals with reasons for the answers.

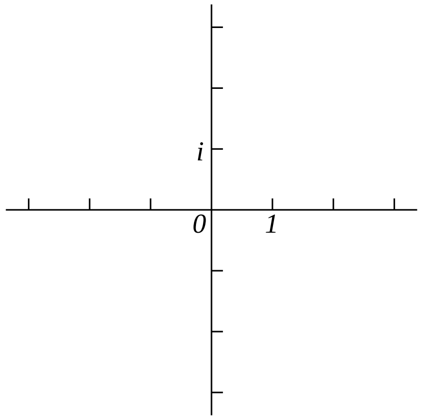
a) $\displaystyle\int_C \frac{1}{z}\,dz.$

b) $\displaystyle\int_C \frac{1}{z^2}\,dz.$

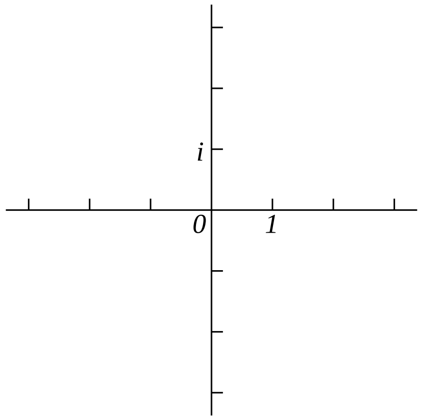
c) $\displaystyle\int_C \frac{e^{5z}}{z}\,dz.$

d) $\displaystyle\int_C \frac{e^{5z}}{z^2}\,dz.$

(8) 6. a) Sketch the region of the complex numbers satisfying the inequalities $1 \leq |z| \leq 3$ and $0 \leq \arg z \leq \dfrac{\pi}{4}$.

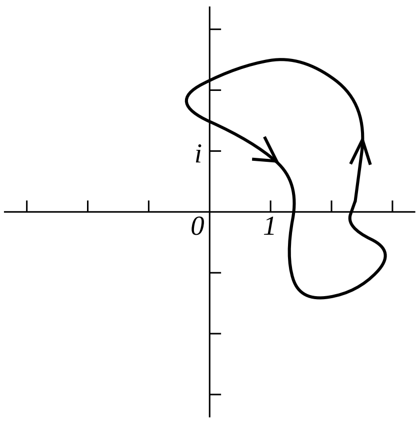


b) Sketch the image of the region in part a) under the mapping $z \to \operatorname{Log} z$, the *principal branch* of logarithm.



c) Compute $\displaystyle\int_C \operatorname{Log} z \, dz$ where C is the curve shown.

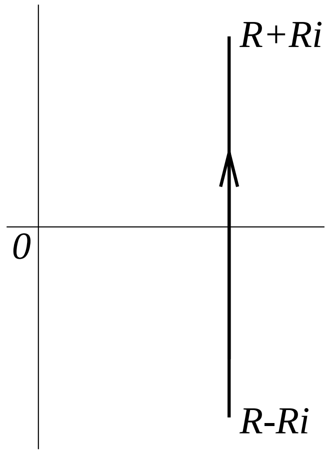
Briefly explain your answer.



(12) 7. Suppose C_R is the vertical line segment from $R - Ri$ to $R + Ri$ as shown, and $f(z) = \dfrac{e^{(-z^2)}}{z^4+1}$.

Prove that $\displaystyle\lim_{R\to+\infty} \int_{C_R} f(z)\, dz = 0$.

Show details of your estimates.



Midterm Exam for Math 403

March 9, 1999

NAME ______________________________________

Do all problems, in any order.
Show your work.
No notes or calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	16	
2	16	
3	16	
4	16	
5	16	
6	8	
7	12	
Total Points Earned:		