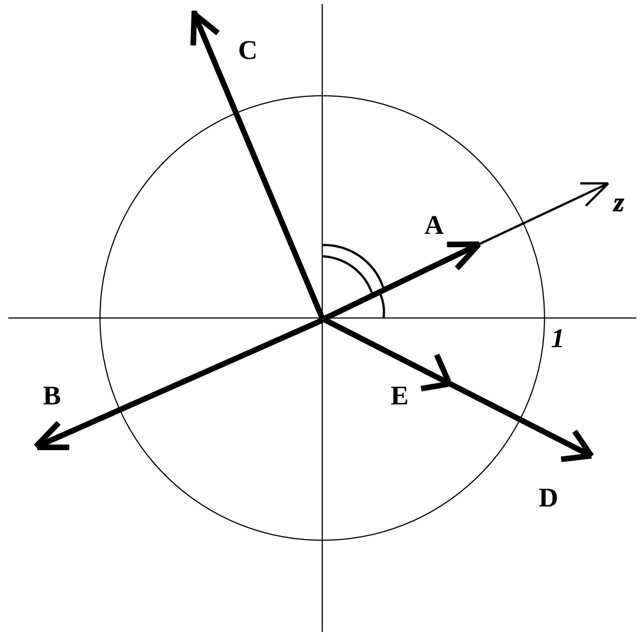
Math 403, section 5 January 23, 2001

How to "locate" some complex numbers

I asked the following: Suppose z is the complex number pictured. Draw and label the following complex numbers, as well as you can:

$$\mathbf{A} = \frac{1}{2}z \qquad \mathbf{B} = -z \qquad \mathbf{C} = iz \qquad \mathbf{D} = \overline{z} \qquad \mathbf{E} = \frac{1}{z}$$

Here are the "answers".



A is a positive real multiple of z, so it has the same direction. Its length is half the length of **A**. **B** is $-z$, the additive vector inverse of z. It just points "backwards" compared to z, with the same magnitude. **C** is the first complex number that needs a bit of thought. What does multiplication "do"? It adds arguments, and it multiplies the lengths of the complex numbers. i is a complex number of modulus 1, so the modulus of iz is the same as the modulus of z. Since i has argument $\frac{\pi}{2}$ (a right angle!), iz is z rotated in the positive, counterclockwise direction by a right angle. Note that in the context of complex variables, you can look back at **B** now: $-z = i \cdot i \cdot z$, so that $-z$ is just z rotated by a right angle and again rotated by a right angle. Of course the answer is the same, but thinking again is a good thing here. **D** is just the "vector" z reflected across the x-axis, the real axis. Complex conjugation reflects across the real axis. (**Question** What's a formula, in complex variables notation, for the result of reflecting z across the y-axis?) To get **E**, one needs the magic (?) formula $\dfrac{1}{z} = \dfrac{\overline{z}}{|z|^2}$. We've already "got" $\overline{z}$ (that's D). We just need to multiply it by the factor $\dfrac{1}{|z|^2}$, which is a positive real number, so **E** shares the same direction as **D**. We can now debate exactly where to draw **E** along the line segment representing $\overline{z}$. My own estimate is that the length of z is more than 1 and less than 1.5. So my guess for **E** is about where I put it. Reasonable people could certainly disagree about where to put **E**, but it should be along the line segment representing **D**, and fairly well inside the unit circle, but not really close to 0.