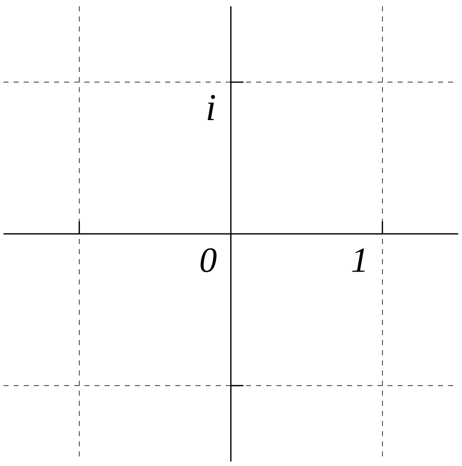
(6) 1. Find the principal argument Arg z when $z = (2 - 2i)^7$.

(12) 2. Describe all solutions of $z^5 = 1$ algebraically in either rectangular or polar form. Sketch these solutions on the axes provided.

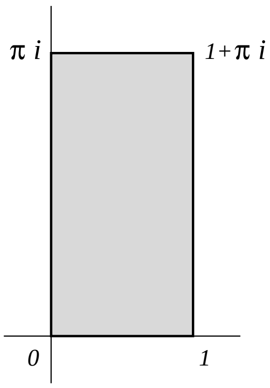


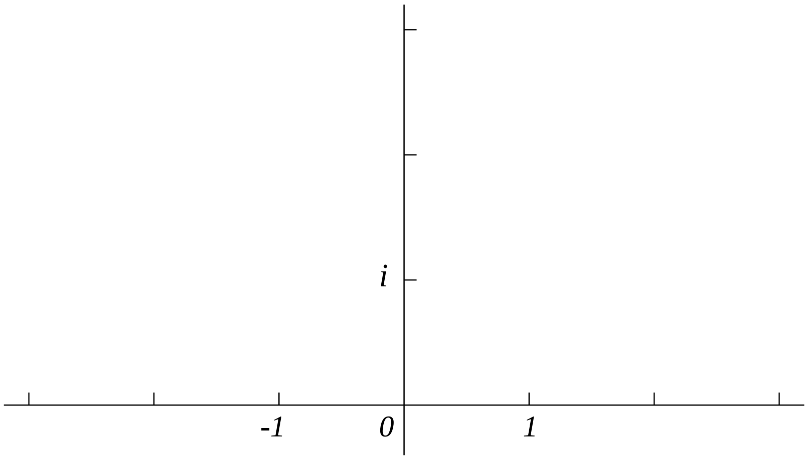
(12) 3. Suppose f is an analytic function with real part $u(x, y)$ and imaginary part $v(x, y)$. Explain why the function

$$H(x, y) = \big(u(x, y)\big)^2 - \big(v(x, y)\big)^2$$

is harmonic, and find a harmonic conjugate of $H(x, y)$ in terms of $u(x, y)$ and $v(x, y)$.

(16) 4. a) Suppose S is the rectangular region whose boundary is the rectangle with corners 0, 1, πi, and $1 + \pi i$ as shown. Sketch the image of S under the exponential mapping on the axes given.





b) Find <u>all</u> solutions of $(e^z)^2 = -1$.

(10) 5. Find the two points in the complex plane where the function $\left(\frac{1}{4}x^4 + y^2\right) + i(x^2y)$ is complex differentiable. Where is this function analytic?

(6) 6. Describe with some justification what happens to $\sin z$ as $z \to \infty$ along the positive imaginary axis.

(12) 7. Describe a connected open subset W of the plane that includes the positive real axis on which an analytic branch of $\log(z^3 + 5)$ can be defined. Justify your assertion with an explanation.

Hint A simple wedge might be a good candidate.

(16) 8. a) Compute $\int_C (x^2 + iy)\, dz$ where C is the straight line segment from 0 to $1 + 2i$.

b) If C is the circle of radius 2 centered at 0, verify that

$$\left| \int_C \left(\frac{z^2 + 3}{5z^3 - 3} \right) dz \right| \leq \frac{28\pi}{37}.$$

Show details of your estimates.

(10) 9. Compute

$$\int_C \left(\frac{7}{z^5} + \frac{4}{z} + 5z^8 \right) dz$$

where C is the boundary of the unit circle.

First Exam for Math 403, section 5

February 28, 2001

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes or calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	6	
2	12	
3	12	
4	16	
5	10	
6	6	
7	12	
8	16	
9	10	
Total Points Earned:		