April 15, 2002

Bald answers to the Review Problems for the second exam in section 1 of Math 403

1. Use the fact that 0 is an essential singularity. (*Why* is 0 an essential singularity?)

2. a) $10\pi i$ b) 0 c) 0.

3. One way: imitate the proof of Liouville's Theorem given in class (that is, use the Cauchy estimates).

4. You can use the Cauchy estimates again. Or you can look at $\frac{h(z)}{z^7}$ near 0 first: investigate this isolated singularity. Then look at the quotient over all of $\mathbb{C}$.

5. $\frac{6}{z^3}+\frac{3}{10z}+\ldots$ is the beginning of the Laurent series. The type of singularity (a pole of order 3) and the residue $\left(\frac{3}{10}\right)$ are on display.

6. Use Rouché's Theorem twice. The closed curves will be two circles centered at 0 of radius 1 and 10, respectively. Choose pieces of $P(z)$ which are "big" on each circle and have roots easy to find and count.

7. Exactly $\sqrt{2}$. Power series converge in discs. The Taylor series of an analytic function converges in the largest disc that it possibly can.

8. $\frac{A}{z}+\frac{B}{z-1}$: find A and B. Then use a geometric series. The coefficient of z^{10} is -2. The coefficient of z^{-10} is 0.

9. Take exp's Taylor series at 0 and stuff in z^2, then multiply. The fourth derivative is 12.

10. `Maple` reports that the answer is $\sqrt{2}\pi$.

11. `Maple` reports that the answer is $\frac{\pi}{4}$.

12. `Maple` reports (really!) that the answer is

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-1/2*Pi*sinh(a)+1/2*a*Pi*cosh(a)+1/2*Pi*cosh(a)-1/2*a*Pi*sinh(a)
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but this can be simplified: $\frac{\pi e^{-a}}{2}(a+1)$.

13. What is $q(0)$? What is the order of the zero?

14. This is a tricky question: $M(z)$ is much nicer than it "looks" and its series may be really nice. Don't try to compute it, though!