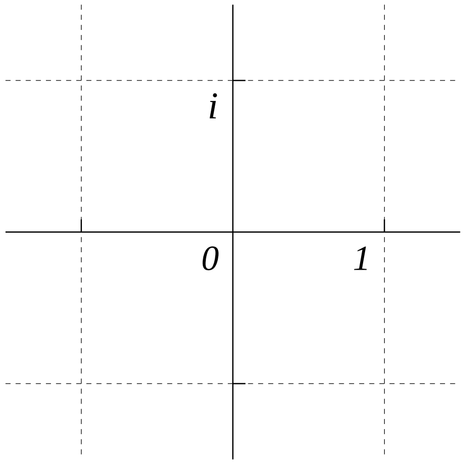
(8) 1. Find the principal argument $\operatorname{Arg} z$ and the modulus $|z|$ if $z = (2 - 2i)^7$.

Comment Your answer(s) should be exact. Answers may use traditional mathematical constants such as π and e and operations involving arithmetic and root extraction.

(10) 2. Describe all solutions of $z^3 = 1$ algebraically in [either] rectangular [or polar] form. Sketch these solutions on the axes provided. *The brackets indicate text that should have been deleted!*

Comment Your answer(s) should be exact. Answers may use traditional mathematical constants such as π and e and operations involving arithmetic and root extraction.



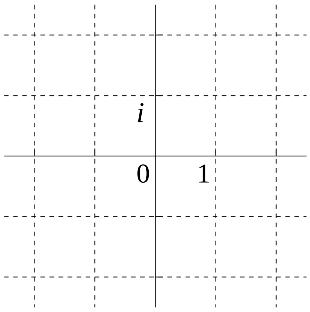
(14) 3. Suppose f is an analytic function with real part $u(x, y)$ and imaginary part $v(x, y)$. Explain why the function

$$H(x, y) = \big(u(x, y)\big)^2 - \big(v(x, y)\big)^2$$

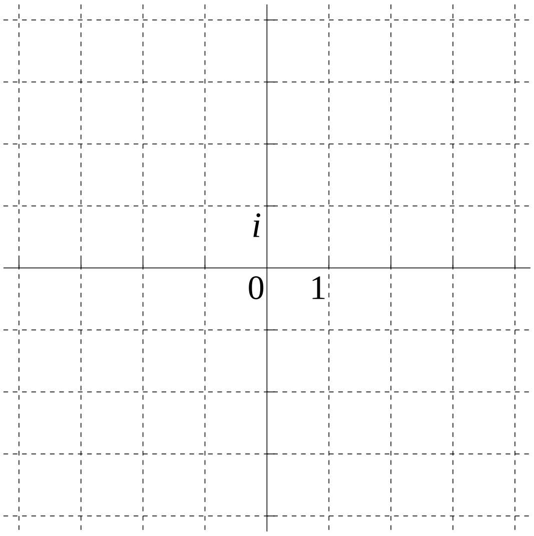
is harmonic, and find a harmonic conjugate of $H(x, y)$ in terms of $u(x, y)$ and $v(x, y)$.

(12) 4. Sketch the region R of z's satisfying the inequalities: $0 < |z| < 2$ and $\frac{\pi}{4} < \operatorname{Arg} z < \frac{\pi}{2}$. What happens to R under the mapping $z \mapsto -z^2$? Sketch the image region carefully, and indicate what happens to the boundary of the region under the mapping. You may want to do this in two stages, first mapping R by $z \mapsto z^2 = w$ and then mapping $w \mapsto -w$.

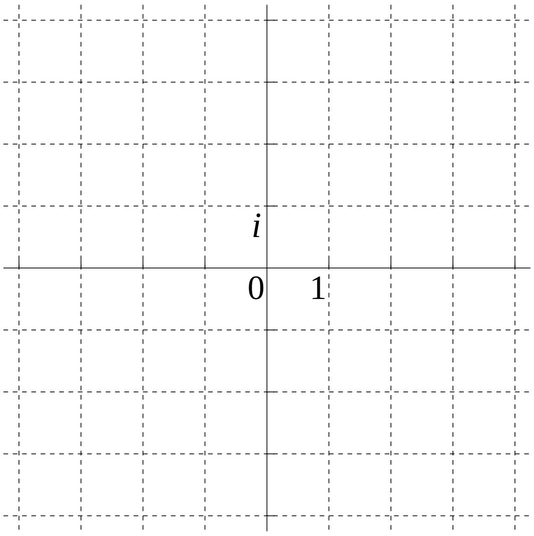
Sketch R here:



Sketch the effect of $z \mapsto z^2 = w$ on R here:



Sketch the effect of $w \mapsto -w$ on the previous answer here:



(10) 5. Find the two points in the complex plane where the function $\left(\frac{1}{4}x^4 + y^2\right) + i(x^2y)$ is complex differentiable. Where is this function analytic?

(6) 6. Describe with some justification what happens to $\sin z$ as $z \to \infty$ along the positive imaginary axis.
(Is it real/imaginary? Bounded/unbounded? Increasing/decreasing in modulus?)

(14) 7. Suppose $S(z)$ is the power series

$$\sum_{n=1}^{\infty} \frac{z^n}{n(2+i)^n}.$$

a) What is the radius of convergence of $S(z)$?

b) Write a power series representing $S'(z)$ inside the radius of convergence of $S(z)$. Find the exact value of $S'(1)$.

(16) 8. a) Compute $\int_C (x^2 + iy)\, dz$ where C is the straight line segment from 0 to $1 + 2i$.

b) If C is the circle of radius 2 centered at 0, verify that

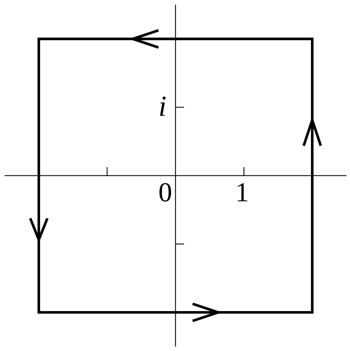
$$\left| \int_C \left(\frac{z^2 + 3}{5z^3 - 3} \right) dz \right| \leq \frac{28\pi}{37}.$$

Show details of your estimates.

(10) 9. Compute

$$\int_W 5z^8 - \frac{2}{z}\,dz$$

where W is the boundary of the square with vertices $-2-2i$, $2-2i$, $2+2i$, and $-2+2i$ as shown.



Comment "An answer alone may not receive full credit" especially here! You should refer to results discussed in class or in the textbook.

First Exam for Math 403, section 1

March 6, 2002

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes or calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	8	
2	10	
3	14	
4	12	
5	10	
6	6	
7	14	
8	16	
9	10	
Total Points Earned:		