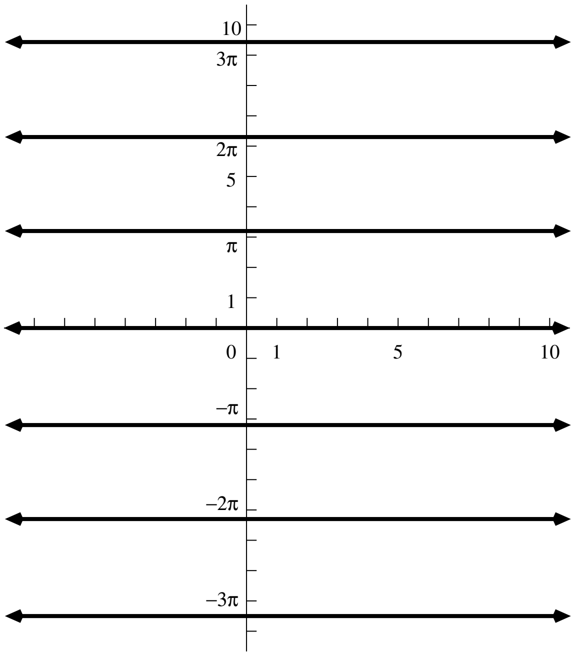
Math 403, section 1 **Answers about exp and log** February 12, 2002

exp

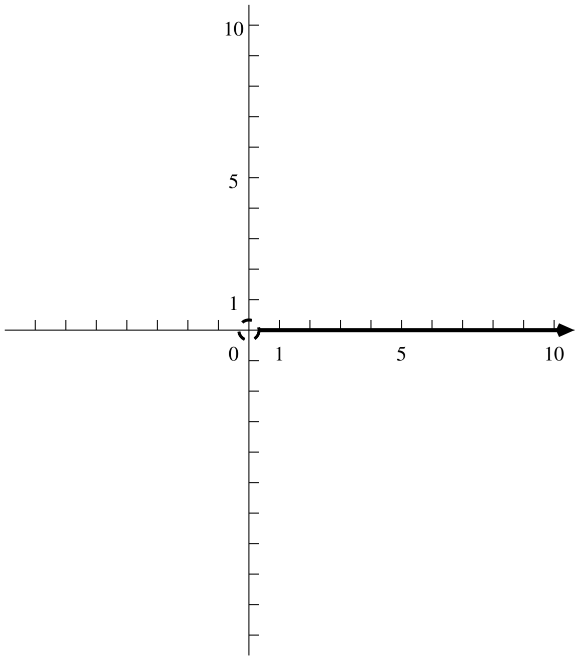
Since $\exp z = e^{\operatorname{Re} z}(\cos(\operatorname{Im} z) + i\sin(\operatorname{Im} z))$, we see that $\exp(1+i)$ must be $e^1((\cos 1)+i(\sin 1))$. Numerical approximations of the numbers are *not* expected.



The complex z's which have $\exp z$ real are just those z's with $\sin(\operatorname{Im} z) = 0$. Of course those are the z's with $\operatorname{Im} z =$ (an integer multiple of πi): an infinite number of horizontal lines. The values of exp are alternately positive and negative real numbers on these lines. Parts of several of the lines are shown.

log

Since $\log z = \ln|z| + i\arg z$ for $z \neq 0$, we see that $\log(1+i)$ must be $\ln\sqrt{2}+i\arg(1+i)$ which is $\ln\sqrt{2} + i(\frac{\pi}{4}) + 2\pi i(\text{any integer})$. Further numerical approximations are *not* expected.



The complex z's which have some value of $\log z$ real are just those z's for which $\arg z$ can be 0. Those z's are the positive real numbers.