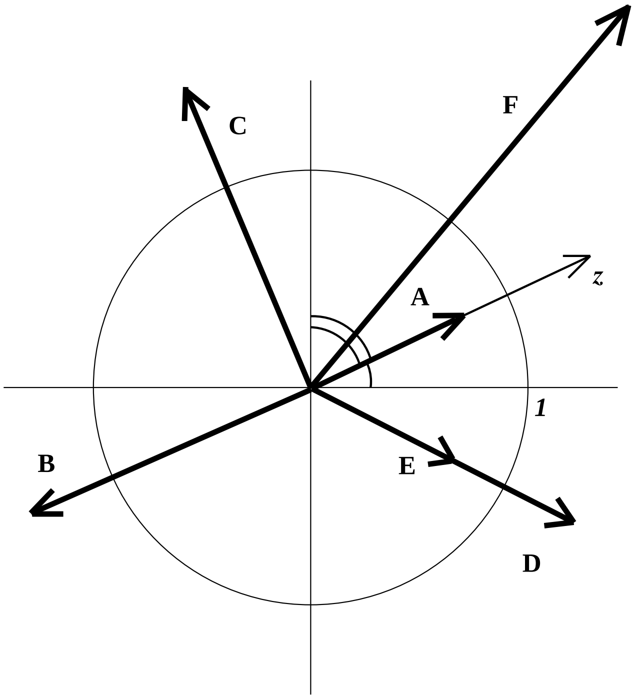
403:01 **How to "locate" some complex numbers** 1/29/2002

I asked the following: Suppose z is the complex number pictured. Draw and label the following complex numbers as well as you can:

$$\mathbf{A} = \tfrac{1}{2}z \qquad \mathbf{B} = -z \qquad \mathbf{C} = iz \qquad \mathbf{D} = \overline{z} \qquad \mathbf{E} = \tfrac{1}{z}$$

Here are the "answers".



A is a positive real multiple of z, so it has the same direction. Its length is half the length of **A**. **B** is $-z$, the additive vector inverse of z. It points "backwards" compared to z, with the same magnitude. **C** is the first complex number needing thought. What is the effect of multiplication? Arguments are added, and the lengths of complex numbers are multiplied. i is a complex number of modulus 1, so the modulus of iz is the same as the modulus of z. Since i has argument $\frac{\pi}{2}$ (a right angle!), iz is z rotated in the positive, counterclockwise direction by a right angle. In the context of complex variables, you can reconsider **B**: $-z = i \cdot i \cdot z$, so $-z$ is just z rotated by a right angle and again rotated by a right angle. The answer's the same, but thinking again is good. **D** is the "vector" z reflected across the x-axis, the real axis. Complex conjugation reflects across the real axis. (**Question** What's a formula, in complex variables notation, for the result of reflecting z across the y-axis?) To get **E**, use the formula $\frac{1}{z} = \frac{\overline{z}}{|z|^2}$. We've already "got" $\overline{z}$ (that's **D**). We must multiply **D** by $\frac{1}{|z|^2}$, a positive real number, so **E** has the same direction as **D**. We can debate exactly where to draw **E** along the line segment representing $\overline{z}$. I estimate the length of z is more than 1 and less than 1.5. So my guess for **E** is about where it is drawn. Certainly **E**'s precise position isn't clear, but it should be along the line segment representing **D**, well inside the unit circle, but not really close to 0. **F**'s argument is twice z's, and its modulus is the square of z's. I think the modulus of **F** is between 2 and 2.25 which is what I've tried to draw.