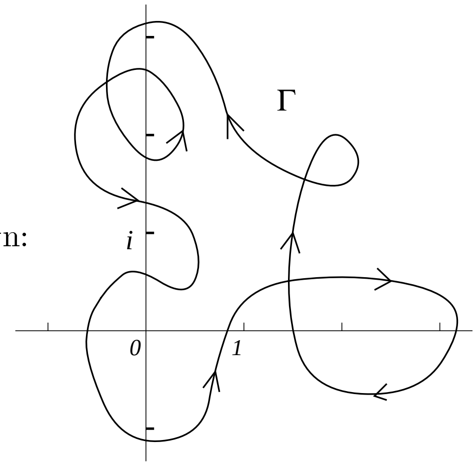
December 2, 1997

Preparation for the final exam

The purpose of the exam is to assess your knowledge of complex variables and to prepare you for our written qualifying exam. The final exam will be given for two consecutive hours on Wednesday, December 17, with time and place to be announced in class. If you can't be there, please arrange an alternate time and place with me as soon as possible. The exam will consist of at most 6 questions. At least 3 of the 6 questions will be taken from the questions written below. No notes or texts should be used on the exam.

1 Suppose $\mathbb{C}^*$ is $\mathbb{C}\backslash\{0\}$ and $\triangle^*$ is $\triangle(0,1)\backslash\{0\}$ and A is the annulus $\{z \in \mathbb{C} : 1 < |z| < 2\}$. It is known that each of these are homeomorphic to the others (indeed, real analytically homeomorphic!). Prove that no two of these are biholomorphic.

2 Compute $\displaystyle\int_\Gamma \frac{z^3}{(e^{2\pi i z}-1)^2}\,dz$ where Γ is the closed curve shown:



3 Consider the polynomial $p(z) = z^5 + z^3 + 5z^2 + 2$. How many zeros (counting multiplicity) does p have in the annular region $1 < |z| < 2$?

4 Let f be a complex-valued function in the open unit disc, $\triangle = \triangle(0,1)$, such that the functions $g = f^2$ and $h = f^3$ are both holomorphic. Prove that f is holomorphic in $\triangle$.

Suppose you only know that f^2 is holomorphic. Does it follow that f is holomorphic?

5 Suppose that U is a connected open subset of $\mathbb{C}$, and that S is a discrete subset of U. Prove that $U\backslash S$ is also a connected open subset of $\mathbb{C}$.

6 Prove that a positive harmonic function on $\mathbb{R}^2$ is constant.