Experimental Math: surely one of the top 203 Byrne Seminars ever given!

More searches for solutions

We will follow Mr. Rowland's presentation with some questions for groups of students. The procedures which were used today are on the web page for the Seventh Meeting of the seminar. Copy and change them to help you investigate the questions which follow.

Rice

Search for positive integer solutions to the equation $x^2 + y^2 + z^2 = w^2$. Are there any? Eliminate any duplicates and solutions that are not primitive. What can you say about these solutions? In particular, which variables must be odd and which must be even?

Wheat

We looked at the equations $x^2 + y^2 = z^2$ and $x^2 + y^2 = 2z^2$ and $x^2 + y^2 = 3z^2$. The number of positive integer solutions seems to change based on the multiplier of z^2. Begin to check systematically. That is, look at $x^2 + y^2 = Qz^2$ when $Q = 4$, $Q = 5$, and so on. For some Q's, the equation has solutions, and for other Q's it doesn't seem to have solutions. Accumulate some evidence. You may then want to consult the *The On-Line Encyclopedia of Integer Sequences* and see if you can get some insight.

Corn

There certainly seem to be lots of Pythagorean triples. What if we start putting extra conditions on them?

• ***y* close to *z***

Suppose we want positive integer solutions to $x^2 + y^2 = z^2$ but look for y's which are close to the corresponding z's. You can search for ...

Solutions with $y = z - 1$; solutions with $y = z - 2$; solutions with $y = z - 3$; etc.

• ***x* close to *y***

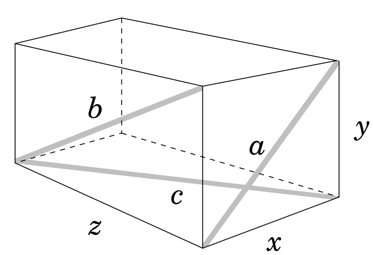
Suppose we want positive integer solutions to $x^2 + y^2 = z^2$ but look for x's which are close to the corresponding y's. You can search for ...

Solutions with $x = y + 1$; solutions with $x = y + 2$; solutions with $x = y + 3$; etc.

Barley

Suppose we have a rectangular box with positive integer side lengths x, y, and z. The sides of this box have many different diagonals but they only come in 3 different lengths, say a, b, and c. A drawing of such a box is shown to the right. Can all of the six values (x, y, z, a, b, and c) be positive integers? So we want positive integer solutions to these equations:

$$x^2 + y^2 = a^2; \quad y^2 + z^2 = b^2; \quad z^2 + x^2 = c^2.$$



Solutions to this system are called *Euler bricks*. Please search for solutions.

Comment If we also want the box's main diagonal (the "space diagonal" connecting the most distant corners) to be an integer – that is, $x^2 + y^2 + z^2 = d^2$ where d is an integer – then no solution is known. It's not known if there is *any solution*, even one not yet found!