(15) 1. Suppose L_1 is the straight line described parametrically by $\begin{cases} x = 3t+1 \\ y = 7t+2 \\ z = -t+1 \end{cases}$ and L_2 is the straight line described parametrically by $\begin{cases} x = 2s-2 \\ y = s+6 \\ z = -2s+6 \end{cases}$.

a) The lines L_1 and L_2 intersect in a point. Find the point.

b) Find an equation for the plane containing L_1 and L_2.

(15) 2. Suppose $f(x, y, z) = x^2 e^{3y-z}$.

a) Compute ∇f.

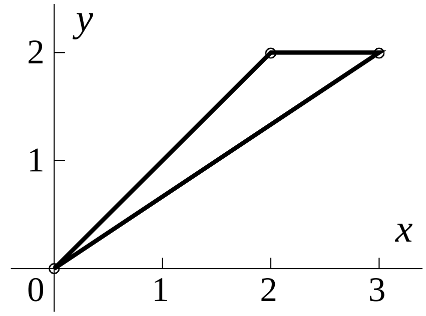
b) Suppose $p = (2, 1, 3)$. Find the maximum rate of change of f at p and find a vector in the direction of this maximum increase.

Maximum rate of change: ________________

Vector in the direction of that rate: ________________

c) Write parametric equations for the line normal to the level surface of f at p.

(15) 3. Compute $\displaystyle\iint_D x^4 y\, dA$ where D is the triangular region in the plane whose vertices are at $(0,0)$, $(2,2)$, and $(3,2)$.



(20) 4. Suppose $J = \int_0^1 \int_0^\pi \int_0^{\sqrt{1-z}} \left(z^2 \sin\theta\right)\, r dr\, d\theta\, dz$, a triple integral in cylindrical coordinates.

a) Sketch the solid region in $\mathbb{R}^3$ over which the integral J is computed, and accompany your sketch with equations describing the boundary surfaces of the region using the standard rectangular coordinates x, y, and z.

b) Compute J.

(15) 5. Suppose $f(x, y) = (y+3)e^{(x^2-y)}$.

a) $f(x, y)$ has one critical point. Find this point.

You must check that you have found all the critical points, and that there is exactly one.

b) Use the Second Derivative Test to determine the nature of the one critical point.

(25) 6. Find the total flux upward through the upper hemisphere (where $z \geq 0$) of the sphere

$$x^2 + y^2 + z^2 = a^2$$

of the vector field

$$\mathbf{T}(x, y, z) = \left(\frac{x^3}{3}\right)\mathbf{i} + \left(yz^2 + e^{\sqrt{zx}}\right)\mathbf{j} + \left(zy^2 + y + 2 + \sin\left(x^3\right)\right)\mathbf{k}$$

Note Do *not* compute this directly! Use the Divergence Theorem on some "simple" solid to change the desired computation to the computation of a triple integral and a simpler flux integral. Evaluate those integrals, taking as much advantage of symmetry as possible.

(20) 7. Suppose $\mathbf{F}(x, y, z) = \left(y^2 z\right)\mathbf{i} + (2xyz)\,\mathbf{j} + \left(xy^2 + 4z\right)\mathbf{k}$, a vector field defined and continuously differentiable throughout space.

a) Determine whether there is a scalar function $P(x, y, z)$ defined everywhere in space such that $\nabla P = \mathbf{F}$. If there is such a P, find it; if there is not, explain why not.

b) Compute the integral $\displaystyle\int_C \mathbf{F} \cdot \mathbf{t}\, ds$, where C is the circular helix whose position vector is given by $\mathbf{R}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j} + t\,\mathbf{k}$ for $0 \leq t \leq 2\pi$. Use information from your answer to a) to help if you wish.

(15) 8. Evaluate the line integral

$$\int_R \left(2xy^2 + \cos x\right) dx + \left(-3x^2 - e^y\right) dy$$

where R is the rectangle with vertices $(0,1)$, $(2,1)$, $(2,2)$, and $(0,2)$ oriented in the usual (counterclockwise) fashion.

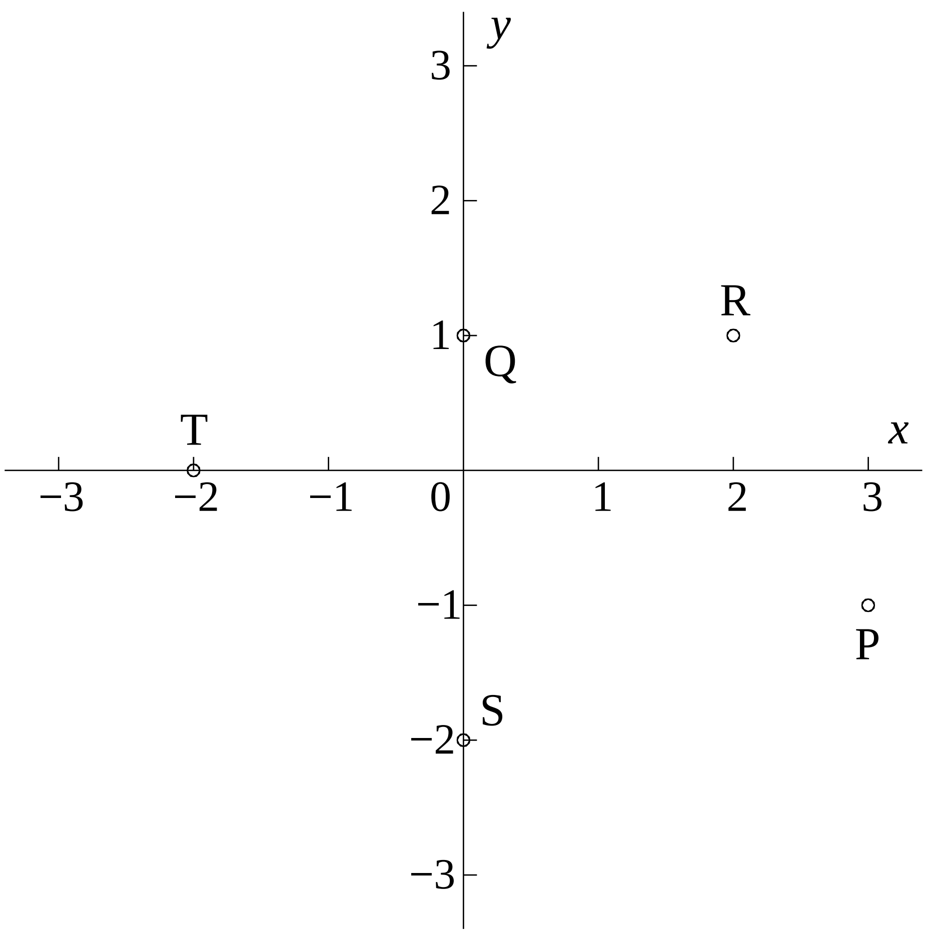
Note Use *any* method to evaluate this integral. Some methods are easier than others.

(20) 9. Sketch the three level curves of the function

$$W(x, y) = x - y^2$$

which pass through the points $P = (3, -1)$ and $Q = (0, 1)$ and $R = (2, 1)$. Be sure to label each curve with the appropriate function value and be sure that your drawing is clear and unambiguous.

Also sketch on the same axis the vectors of the gradient vector field ∇W at the points P and Q and R and S and T. The point $S = (0, -2)$ and $T = (-2, 0)$.



(20) 10. Suppose R is the bounded region in the xy-plane defined by $y = x^2$ and $y = x + 2$.

a) Sketch the region R.

b) If $f(x, y)$ is a function defined in the region R, describe how to write $\int\int_R f(x, y)\, dA$ as a sum of one or more iterated $dx\, dy$ integrals.

c) If $f(x, y)$ is a function defined in the region R, describe how to write $\int\int_R f(x, y)\, dA$ as a sum of one or more iterated $dy\, dx$ integrals.

(12) 11. Find the curvature of the ellipse $\begin{cases} x = 3\cos t \\ y = 4\sin t \end{cases}$ at the points $(3, 0)$ and $(0, 4)$.

(8) 12. Suppose z is defined implicitly as a function of x and y by the equation

$$z^2y + 3xy^2 + e^{(x^2z)} = 1.$$

Find $\dfrac{\partial z}{\partial x}$.

A A

Final Exam for Math 251, sections 5–10

May, 2006

NAME ____

Please circle your section number: 5 6 7 8 9 10

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	15	
2	15	
3	15	
4	20	
5	15	
6	25	
7	20	
8	15	
9	20	
10	20	
11	12	
12	8	
Total Points Earned:		

A A