(20)   1. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\lim\limits_{x \to -2} \dfrac{x^2 - x - 6}{x + 2}$

b) $\lim\limits_{x \to 0} \dfrac{\sqrt{3 + x} - \sqrt{3}}{x}$

c) $\lim\limits_{x \to 1^-} \dfrac{3x + \ln x}{x}$

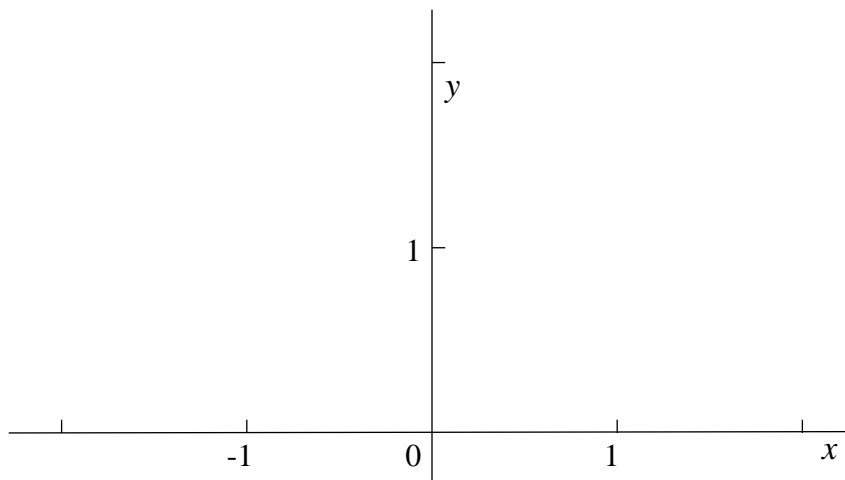d) $\lim\limits_{x \to 0^+} \dfrac{2}{\sin x}$

(14)   2. Suppose that the function F is described by

$$F(x) = \begin{cases} \cos x & \text{if } -\frac{\pi}{2} \le x \le 0 \\ Ax + B & \text{if } 0 < x \le 1 \\ x - 1 & \text{if } 1 < x \le 2 \end{cases} .$$

a) Find A and B so that F is continuous for all numbers in its domain. Briefly explain your answer.

b) Graph $y = F(x)$ on the axes given for the values of A and B found in a).

(12)   3. A ladder which is 12 feet long has one end on flat ground and the other end on the vertical wall of a building. H is the height from the ground to the point at which the ladder touches the building. D is the distance between the bottom of the ladder and the bottom of the wall. θ is the acute angle between the ladder and the ground.
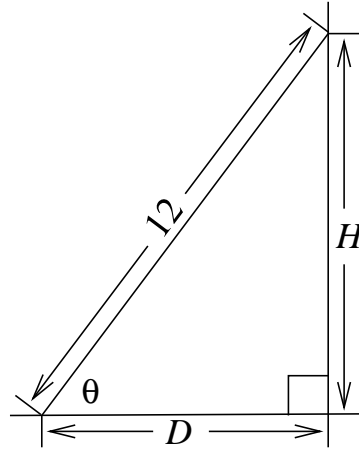
a) Write H as a function of D: that is, a formula involving D and no other <u>variable</u>. What is the domain of this function?

b) Write H as a function of θ: that is, a formula involving θ and no other <u>variable</u>. What is the domain of this function?

(12)   4. a) Write the definition of derivative as a limit and *use this definition* to find the derivative
of $f(x) = \dfrac{2}{x+3}$.

b) Use your answer to a) to find an equation for the line tangent to the curve $y = \dfrac{2}{x+3}$
when $x = 2$.

(9)   5. Do *not* algebraically simplify the answers in this problem!

a) If $f(x) = -8x^7 + 4x^4 + 24$, what is $f'(x)$?

b) If $g(x) = \dfrac{x^2 + 1}{2e^x - 3}$, what is $g'(x)$?

c) If $h(x) = \sqrt{3 - \sin x}$, what is $h'(x)$?

(8)   6. k is a differentiable function and the following is known about k and its derivatives.

$$k(1) = 2 \quad k'(1) = -3 \quad k''(1) = 4$$

Suppose that $j(x) = k(e^x)$ (this is a composition). Compute $j(0)$ and $j'(0)$ and $j''(0)$. Give the exact answer in each case.

(13) 7. Below is the graph of a differentiable function, Q, whose domain is $-2 < x < 6$. Use this graph to answer the following questions as accurately as you can.
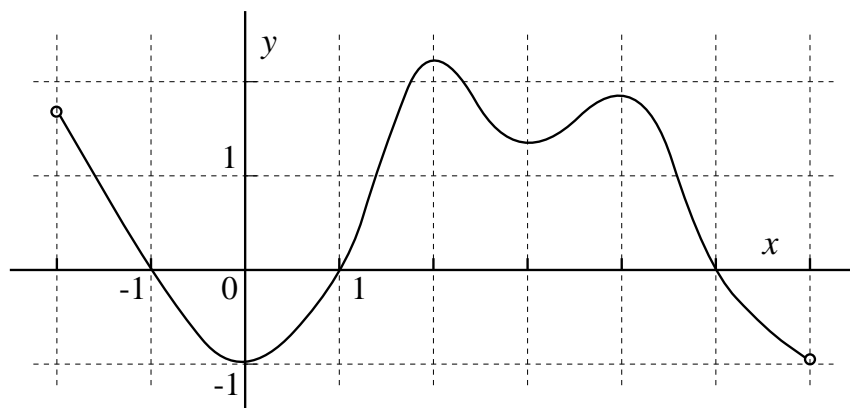

The graph of $y = Q(x)$

a) For which x's is $Q(x) = 0$?

b) What are the largest intervals on which $Q(x) > 0$ for all x's in each interval?

c) For which x's is $Q'(x) = 0$?

d) What are the largest intervals on which $Q'(x) > 0$ for all x's in each interval?

(12) 8. The line $y = \frac{1}{2}x + 3$ is tangent to the parabola $y = x^2 + B$. Use algebra and calculus to find the exact value of B.

Hint: the two formulas are equal at the tangent point, and the line is tangent to the curve at that point.
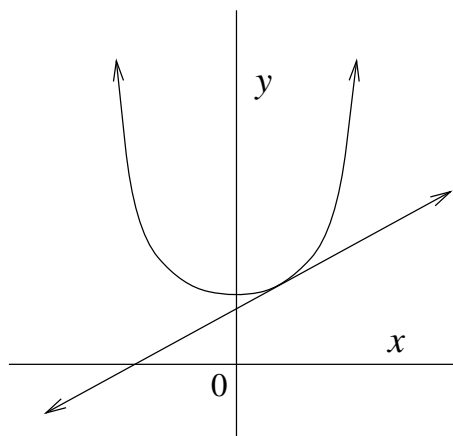
Exam 1 for Math 135

Sections 1, 2, and 4

October 12, 1998

NAME (*please print*): _____

SIGNATURE: _____

SECTION #: _____

Do all problems, in any order.

Show all your work. Full credit may not be given for an answer alone.

You may use <u>one</u> sheet of notes and any standard calculator without a QWERTY keypad on this exam or symbolic manipulation capability.

You may use <u>no</u> other materials.

Problem Number	Possible Points	Points Earned:
1	20	
2	14	
3	12	
4	12	
5	9	
6	8	
7	13	
8	12	
Total Points Earned:		