(20) 1. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$

b) $\lim\limits_{x \to -2} \dfrac{\frac{1}{x} + \frac{1}{2}}{x + 2}$

c) $\lim\limits_{x \to \frac{\pi}{2}^-} \dfrac{\sin x}{x - \frac{\pi}{2}}$

d) $\lim\limits_{x \to 0^+} \dfrac{|x|}{x}$

(14)  2. a) Suppose $f(x)$ is a function. Write the **definition** of $f'(x)$, the derivative of $f(x)$, as a limit.

b) Use your answer to a) and familiar properties of limits to find the derivative of $f(x) = \sqrt{x-3}$.
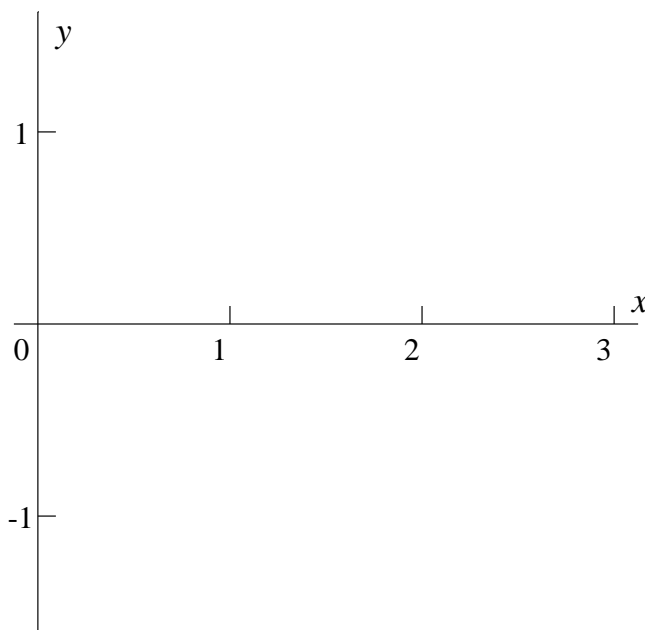
c) Use your answer to b) to write an equation for the line tangent to $y = \sqrt{x-3}$ when $x = 7$.
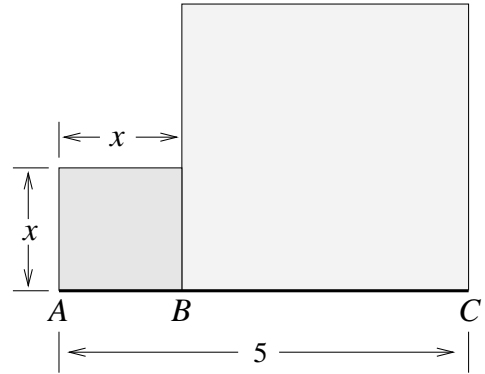
(14)   3. Suppose that the function $G$ is described by

$$G(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ Ax^2 + B & \text{if } 1 < x \leq 2 \\ x - 3 & \text{if } 2 < x \leq 3 \end{cases}.$$

a) Find $A$ and $B$ so that $G$ is continuous for all numbers in its domain. Briefly explain your answer.

b) Graph $y = G(x)$ on the axes given for the values of $A$ and $B$ found in a).
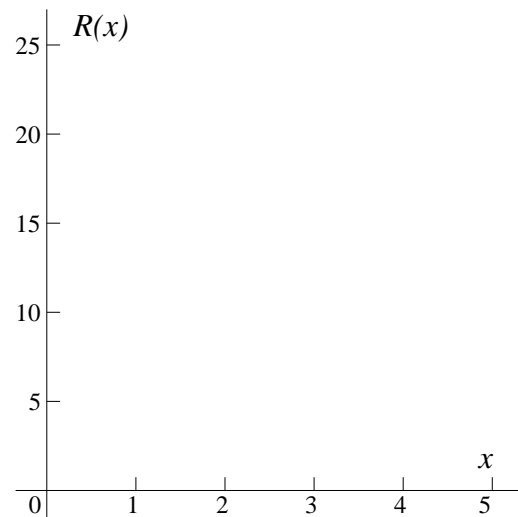
(16)    4. The line segment $\overline{AC}$ is 5 units long and is divided
into two parts by the point $B$. Suppose that $x$ is the
length of the segment $\overline{AB}$ and that $R(x)$ is the sum
of the areas of the two squares with sides $\overline{AB}$ and $\overline{BC}$
as shown.

a) Write an algebraic formula for $R(x)$ as a function of $x$.

b) Graph $R(x)$ on the axes shown. Note that the
horizontal and vertical axes have different scales.

c) Use algebra to find all <u>exact</u> values of the length $x$ for which $R(x) = 17$.

(9)    5. Use all differentiation rules here. Please do *not* simplify the answers in this problem!

a) If $P(x) = 6x^5 - 3x^3 + 4$, what is $P'(x)$?

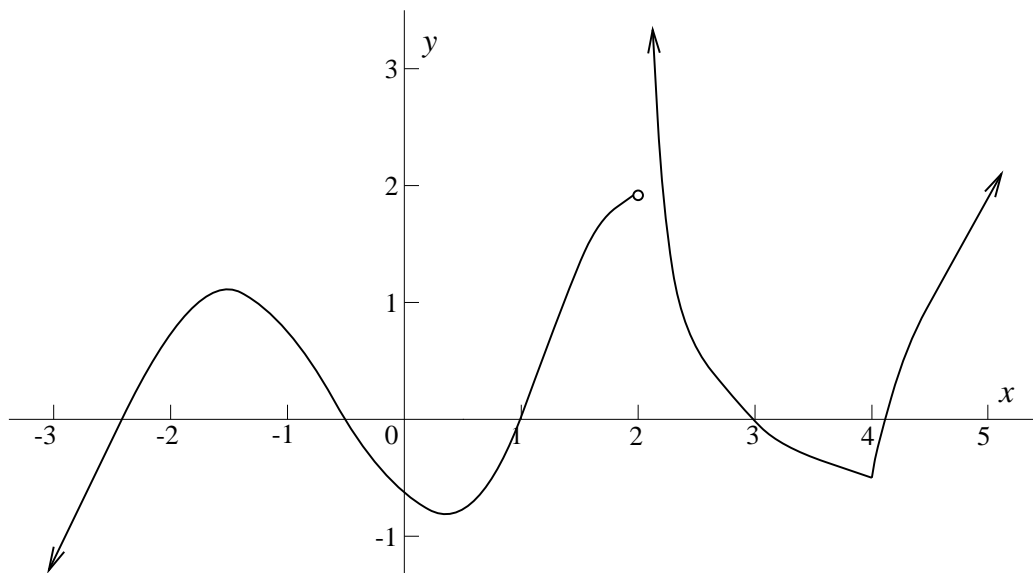b) If $Q(x) = \dfrac{x^3 + 1}{3x - 2}$, what is $Q'(x)$?

c) If $R(x) = (5 + \cos x)^{100}$, what is $R'(x)$?

(8)    6. Suppose that $T$ is a differentiable function and the following facts are known about $T$ and its derivatives:
$$T(5) = 0 \text{ and } T'(5) = -3 \text{ and } T''(5) = 2\,.$$
Suppose that $S(x) = e^{T(x)}$ (this is a composition). Compute $S(5)$ and $S'(5)$ and $S''(5)$. Give the exact answer in each case.

(19)    7. Below is a graph of the function $F$. The domain of $F$ is <u>all numbers not equal to 2</u>.



The graph of $y = F(x)$

Use this graph to answer the questions below as accurately as possible.

a) For which $x$ is $F(x) = 0$?

**ANSWER** _____

b) For which $x$ is $F(x) > 0$?

**ANSWER** _____

c) What is $\lim\limits_{x \to 2^-} F(x)$?

**ANSWER** _____

d) What is $\lim\limits_{x \to 2^+} F(x)$?

**ANSWER** _____

e) For which $x$ (in its domain) is $F$ <u>not</u> differentiable?

**ANSWER** _____

f) For which $x$ is $F'(x) = 0$?

**ANSWER** _____

g) What is $F(x)$ for those $x$'s for which $F'(x) = 0$?

**ANSWER** _____

h) For which $x$ is $F'(x) > 0$?

**ANSWER** _____

# Exam 1 for Math 135

## Sections 8, 9, and 10

February 23, 1999

NAME (*please print*): _____

SIGNATURE: _____

SECTION #: _____

**Do all problems, in any order.**

**Show all your work. Full credit may not be given for an answer alone.**

**You may use <u>one</u> sheet of notes and any standard calculator without a QWERTY keypad on this exam or symbolic manipulation capability.**

**You may use <u>no</u> other materials.**

| Problem Number | Possible Points | Points Earned: |
|:---:|:---:|:---:|
| 1 | 20 | |
| 2 | 14 | |
| 3 | 14 | |
| 4 | 16 | |
| 5 | 9 | |
| 6 | 8 | |
| 7 | 19 | |
| Total Points Earned: | | |