Some Math 135 review problems for the first half of the course

These problems are mostly from previous Math 135 finals. Almost any Math 135 final will have problems covering this material. Students who are prepared to do problems of this type successfully should succeed on the final. But I'm *not* writing the final, and personal "style" does alter problem selection and presentation.

Definition of derivative

1. Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = x^2 - x$.
2. Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = \frac{1}{3x+4}$.
3. Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = 4 + 3x^2$.
4. Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = \sqrt{1-2x}$.

Computations of limits

1. Evaluate the limits exactly. Give brief evidence supporting your answers which is not based on a calculator graph or calculator computations.
a) $\lim\limits_{x\to\infty} \frac{2x^2-5}{3x^2+1}$ b) $\lim\limits_{x\to 4} \frac{x-4}{\sqrt{x}-2}$ c) $\lim\limits_{x\to 5} \frac{x^2-3x-10}{x-5}$ d) $\lim\limits_{x\to+\infty} \frac{2x^3-x+3}{x^3+2}$ e) $\lim\limits_{x\to 2^-} \frac{x^2-4}{|2-x|}$ f) $\lim\limits_{x\to 2^+} \frac{x^2-4}{|2-x|}$
2. Find the equations of all vertical and horizontal asymptotes of $f(x) = \frac{\sqrt{x^2+4}}{x-3}$.
3. Find the equations of all vertical and horizontal asymptotes of $f(x) = \frac{3+5e^{-2x}}{7-e^{-2x}}$. Computations with exp and log should be simplified as much as possible; approximations are not acceptable.

Computations of derivatives

1. Find $\frac{dy}{dx}$.
a) $y = \frac{2x^2+5}{5x^3+1}$ b) $y = e^{-\sqrt{x^2-1}}$ c) $xy^3 = \cos(7x+5y)$ d) $xe^y = \sin(xy)$ e) $y = x\ln(3x+5)$
f) $2x^3+5x^2y+y^3 = 2$ g) $y = \ln(x^4+3x+1)$ h) $y = \frac{3x^3-1}{3x^4+1}$ i) $y = x^5\tan(3x)$ j) $y = (4x+3)\sqrt{x^3+7}$
2. Suppose $f(x)$ is a differentiable function satisfying $f'(1) = 3$ and $f'\left(\frac{\pi}{4}\right) = -2$, If $g(x) = f(\tan x)$, find $g'\left(\frac{\pi}{4}\right)$.
3. An equation of the tangent line to $y = f(x)$ when $x = 1$ is known to be $2x + y + 3 = 0$. Find $f(1)$ and $f'(1)$.
4. Suppose U is a differentiable function with $U(8) = 5$ and $U'(8) = 3$, and that $V(x) = U(x^3)$. What are $V(2)$ and $V'(2)$? Use this information to write an equation of the tangent line to $y = V(x)$ when $x = 2$.
5. A function is defined implicitly by the equation $x^2 - y^2 - 5xy + x + y = 10$. Find y' in terms of x and y. Find an equation for the line tangent to the graph of this function at the point $(1, 2)$.

Log/exp etc.

1. Find the range of $f(x) = e^{-2x} + e^{3x}$.

2. The rate of bacterial growth at any time is directly proportional to the number of bacteria present at that time. If the population of a certain kind of bacteria doubles in 4 hours and there are 500 bacteria at 1 AM, how many bacteria will there be at 5:30 AM?

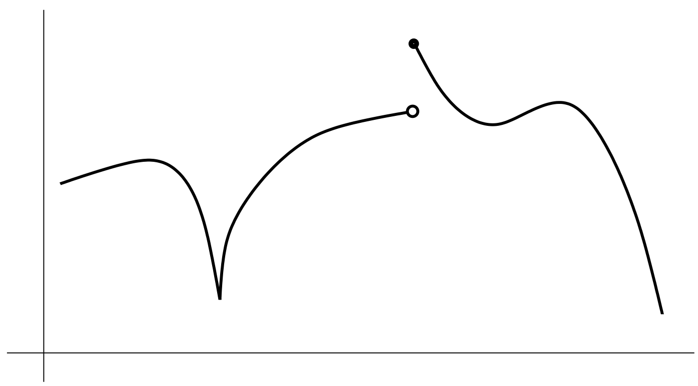
Continuity & differentiability

1. Here $f(x) = \begin{cases} x+3 & \text{if } x \le -2 \\ \frac{1}{2}x^2 + A & \text{if } -2 < x \end{cases}$ where A is a constant to be determined. Find A so that $f(x)$ is continuous for all values of x. Sketch a graph of $y = f(x)$ using that value of A for $-4 \le x \le 2$. Is $f(x)$ differentiable at $x = -2$ using that value of A?

2. Here $f(x) = \begin{cases} Ax^2 - 1 & \text{if } x < -1 \\ x + B & \text{if } -1 \le x \le 1 \\ 2 & \text{if } 1 < x \end{cases}$ where A and B are constants to be determined. Find numbers A and B so that $f(x)$ is continuous for all values of x. Sketch a graph of $y = f(x)$ for $-3 \le x \le 3$.

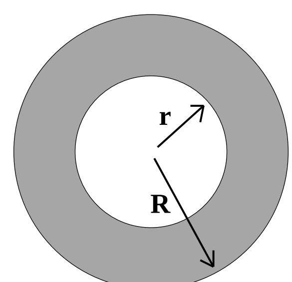
3. In this problem $f(x) = \begin{cases} 1 + x^2 & \text{if } x < 2 \\ A + Bx & \text{if } -2 \le x < 1 \\ x^2 & \text{if } x \ge 1 \end{cases}$. Find A and B so that $f(x)$ is continuous at all points. Sketch a graph of $y = f(x)$ for $-3 \le x \le 3$. For which values of x is $f(x)$ *not* differentiable?

4. In the graph of $y = f(x)$ to the right, identify with **m** any point which is a relative minimum; **M** any point which is a relative maximum; **C** any point which is a critical point; **I** any point which is an inflection point; **NC** any point at which $f(x)$ is *not* continuous; and **ND** any point at which $f(x)$ is *not* differentiable. Some points may have more than one label.



Related rates

1. Two circles have the same center. The inner circle has radius r which is increasing at the rate of 3 inches per second. The outer circle has radius R which is increasing at the rate of 2 inches per second. Suppose A is the area of the region *between* the circles. At a certain time, r is 7 inches and R is 10 inches. What is A at that time? How fast is A changing at that time? Is A increasing or decreasing at that time?



2. Suppose that the right triangle with height h and base b as shown here *always* has an area of 6 square inches. At a certain time, the length of b is 3 inches and it is increasing at the rate of .04 inches per minute. What is the length of h at that time? How fast is the length of h changing at that time? Is it increasing or decreasing? What is the angle θ at that time? How fast is the angle θ changing at that time? Is it increasing or decreasing?