Some Math 135 review problems for the second half of the course

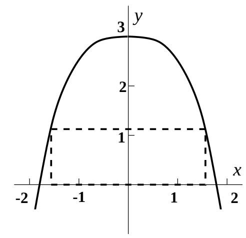
These problems are mostly from previous Math 135 finals. Almost any Math 135 final will have problems covering this material. Students who are prepared to do problems of this type successfully should succeed on the final. But I'm *not* writing the final, and personal "style" does alter problem selection and presentation.

Linear approximation

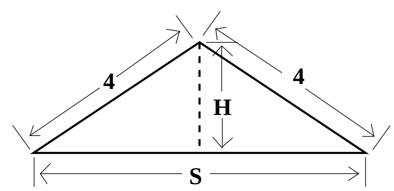
1. Suppose you know that $f(5)=7$, $f'(5)=2$, $g(7)=3$, and $g'(7)=8$. If $F(x)=g(f(x))$, compute $F(5)$ and $F'(5)$. Use linear approximation or differentials to get an approximate value of $F(5.02)$.
2. Suppose $f(x)=\tan(x^2)$. Use linear approximation or differentials to find an approximate value of $f\left(\sqrt{\frac{\pi}{4}}-.03\right)$.

Optimization

1. A box with an open top is to be made from a rectangular sheet of cardboard 5 inches by 8 inches by cutting equal squares out of the four corners and bending up the resulting four flaps to make the sides of the box. Use calculus to find the largest volume of the box. Be sure to explain briefly why your answer gives a maximum.
2. Find the largest and smallest values of $f(x)=30x^3-90x^2+90x+100$ when $2\le x\le 3$. Justify your answer with calculus.
3. A rectangle is inscribed as shown in the parabola $y=3-x^2$. Of all such rectangles, find the dimensions of the one whose area is a maximum. Explain briefly why your answer gives a maximum.



4. A triangle has two equal sides each 4 inches long. What is the length of the third side if the triangle is to have maximum area? Be sure to explain why your answer is a maximum. The diagram may be useful. Use it to express an algebraic relationship between H and $\frac{1}{2}S$ and then compute the area of the triangle.



Intermediate Value Theorem; MVT

1. Suppose $f(x)=\frac{1}{101}x^{101}+\frac{5}{37}x^{37}+\frac{8}{5}x^5+46x+8$. What is $f'(x)$? For which x is $f'(x)$ positive? For which x is $f'(x)$ negative? Describe briefly how calculus can verify that $f(77)$ is bigger than $f(33)$. Do *not* verify this by direct computation of function values.
2. Suppose $M(x)=x^5-7x+4$. Compute $M(2)$ and $M(-2)$ and explain briefly why the results allow you to conclude that $M(x)=0$ has at least one root.

Curve sketching

1. Suppose $f(x)=5x^3-3x^5$. Compute $f'(x)$ and $f''(x)$. Where are each of these functions equal to 0? Find all relative max & min values of $f(x)$, briefly explaining your answers. Find all inflection points of $f(x)$, briefly explaining your answers.
2. In this problem the function G is defined by the formula $G(x)=(2-e^x)(1-e^x)$. Computations with exp and log should be simplified as much as possible; approximations are not acceptable.

a) What is $\lim_{x\to-\infty} G(x)$? What is $\lim_{x\to+\infty} G(x)$?

b) For which x is $G(x)=0$?

This problem continues on the next page.

c) Compute $G'(x)$. For which x is $G'(x) = 0$?
d) Compute $G''(x)$. For which x is $G''(x) = 0$?
e) Sketch a graph of $y = G(x)$, using the information you have obtained in the previous parts of the problem. Indicate precisely any relative maxima or minima or points of inflection with exact coordinates, and any vertical or horizontal asymptotes with exact equations. (Axes with $-5 \leq x \leq 1$ and $0 \leq y \leq 6$ were supplied.)

Antiderivatives & initial value problems

1. Suppose $f''(x) = x + \frac{1}{x^2}$ and $f(1) = 1$ and $f'(1) = -2$. Find a formula for $f(x)$. Computations with exp and log should be simplified as much as possible; approximations are not acceptable.
2. Suppose $f''(x) = \cos x + \sin(2x)$ and $f(0) = 0$ and $f'(\pi) = 0$. Find a formula for $f(x)$. Computations with sine and cosine should be simplified as much as possible; approximations are not acceptable.
3. Suppose $f''(x) = e^x - 1$ and $f(0) = 2$ and $f'(0) = -3$. Find a formula for $f(x)$. Computations with exp and log should be simplified as much as possible; approximations are not acceptable.

Riemann sums

1. Suppose $f(x) = 2x^2 - 1$. Compute the Riemann sum for $f(x)$ on the interval $[-1, 5]$ with partition $\{-1, 2, 4, 5\}$ using the left-hand endpoints as sample points.
2. Suppose $f(x) = \cos x$. Compute the Riemann sum for $f(x)$ on the interval $[0, \frac{3}{2}\pi]$ obtained by partitioning the interval into 6 equal subintervals and using the right-hand endpoints as sample points.

Definite integral

1. Suppose P and Q are constants, and $f(x) = P\sin(7x) + Qx\cos(7x)$. Find specific values of P and Q so that $f'(x) = x\sin(7x)$. Use your answer to evaluate $\int_9^{\pi/7} x\sin(7x)\,dx$.
2. Suppose $f(x) = xe^x - e^x$. Compute $f'(x)$, and use your answer to evaluate $\int_0^1 xe^x\,dx$ exactly.

Area

1. Sketch the region in the plane bounded by $y = 4 - x^2$ and the x-axis. Find the area of this region.
2. Sketch the region in the plane bounded above by $y = 4 - x^4$ and below by $y = 3$. Find the area of this region.
3. Sketch the region in the plane bounded by the x-axis, the line $x = 2$, and the curve $y = \frac{1}{9}x^5$. Find the area of this region.

FTC

1. $\int_1^4 (2x - 5\sqrt{x})\,dx$
2. $\int_1^4 \frac{1-\sqrt{x}}{x}dx$
3. If $f(x) = \int_{-42}^{x} \frac{\sin(t^2)}{1+t^4}dt$, compute $f(-42)$, $f'(0)$, and $f'(\sqrt{\pi})$ exactly.
4. $\int_1^2 \left(3\sqrt{x} - \frac{1}{x^4}\right)dx$

Substitution

1. $\int (3x - 6)e^{3x^3 - 12x}dx$
2. $\int_0^{\sqrt{\ln 7}} xe^{(x^2)}dx$.