Sample! YOUR TEST WILL HAVE 6 PROBLEMS IN 15 MINUTES. *Sample!*

Math 151:4,5,6 9/25/2006

Name ______________________ **Section** ______

First Computational Test (Limits)

SHOW DETAILS (algebra, limit laws) in the space next to each problem. For the experts: do **NOT** use l'Hospital's rule.

HERE ARE EXAMPLES OF ACCEPTABLE SUPPORTING ANSWERS. OTHER ANSWERS MAY ALSO EARN FULL CREDIT.

1. $\lim_{x\to 1} \frac{x^2(x-3)}{2x+3}$

Since $2(1)+3 \neq 0$, the function $\frac{x^2(x-3)}{2x+3}$ is continuous at 1, and therefore the limit can be computed by just "plugging in".

Answer to 1 $-2/5$

2. $\lim_{x\to 2^-} \frac{|3x-6|}{x-2}$

If $x \to 2^-$ then $x < 2$. Then $3x < 6$ so $3x - 6 < 0$. Therefore $|3x-6| = -(3x-6)$ and $\frac{|3x-6|}{x-2} = -\left(\frac{3x-6}{x-2}\right) = -3$.

Answer to 2 -3

3. $\lim_{x\to\infty} \frac{6x-5x^3}{2x^3+9}$

We divide the top and bottom by x^3 so that $\frac{6x-5x^3}{2x^3+9} = \frac{\frac{6}{x^2}-5}{2+\frac{9}{x^3}}$. Now observe that $\frac{6}{x^2}$ and $\frac{9}{x^3}$ both surely $\to 0$ as $x \to \infty$.

Answer to 3 $-5/2$

4. $\lim_{x\to 3^+} \frac{x(x-3)}{\sqrt{x}-\sqrt{3}}$

Multiply top and bottom by $\sqrt{x}+\sqrt{3}$, and recognize that $(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3}) = x-3$. That is: $\frac{x(x-3)}{\sqrt{x}-\sqrt{3}} = \frac{\big(x(x-3)\big)(\sqrt{x}+\sqrt{3})}{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})} = \frac{\big(x(x-3)\big)(\sqrt{x}+\sqrt{3})}{x-3} = x(\sqrt{x}+\sqrt{3})$

Of course, you could also just use $(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3}) = x-3$ to make $\frac{x(x-3)}{\sqrt{x}-\sqrt{3}} = \frac{x(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})}{\sqrt{x}-\sqrt{3}} = x(\sqrt{x}+\sqrt{3})$ etc.*

Answer to 4 $6\sqrt{3}$

OVER

* There is more than one way to flense a feline.

5. $\lim_{x\to+\infty} \sqrt{x^2-x-3}-x$

$\sqrt{x^2-x-3}-x = \dfrac{\left(\sqrt{x^2-x-3}-x\right)\left(\sqrt{x^2-x-3}+x\right)}{\left(\sqrt{x^2-x-3}+x\right)} = \dfrac{\left(x^2-x-3\right)-x^2}{\left(\sqrt{x^2-x-3}+x\right)} =$ $\dfrac{-x-3}{\left(\sqrt{x^2-x-3}+x\right)} = \dfrac{-x-3}{x\left(\sqrt{1-\frac{1}{x}-\frac{3}{x^2}}+1\right)} = \dfrac{-1-\frac{3}{x}}{\sqrt{1-\frac{1}{x}-\frac{3}{x^2}}+1}$ because since $x>0$ as $x\to+\infty$ we know $\sqrt{x^2}=x$. Then I divided by x on the top and the bottom, and used $\frac{1}{x}\to 0$ and $\frac{3}{x^2}\to 0$ as $x\to\infty$.

Answer to 5 $-1/2$

6. $\lim_{x\to-1^+} \dfrac{x(x-2)}{x+1}$

As $x\to-1^+$, $x>-1$ so that $x+1>0$ and $x+1$ is a SMALL POSITIVE number. Also as $x\to-1^+$, $x(x-2)\to-1(-1-2)=3$, a positive number. Therefore the quotient in the limit expression is $\approx \dfrac{3}{\text{SMALL POSITIVE}}$ and this gives the answer shown.

Answer to 6 ∞

7. $\lim_{x\to-\infty} \dfrac{\sqrt{5x^2-9}}{2-5x}$

When $x<0$, $\sqrt{x^2}=-x$. So as $x\to-\infty$, $\sqrt{5x^2-9}=\sqrt{x^2\left(5-\dfrac{9}{x^2}\right)}=-x\sqrt{5-\dfrac{9}{x^2}}$ and therefore $\dfrac{\sqrt{5x^2-9}}{2-5x}=\dfrac{-x\sqrt{5-\frac{9}{x^2}}}{x\left(\frac{2}{x}-5\right)}=-\dfrac{\sqrt{5-\frac{9}{x^2}}}{\frac{2}{x}-5}$. Again, $\dfrac{9}{x^2}\to 0$ and $\dfrac{2}{x}\to 0$ as $x\to-\infty$, and we get the answer shown.

Answer to 7 $1/\sqrt{5}$

8. $\lim_{x\to 0} \dfrac{(x-1)^3+1}{x}$

"Expand" $(x-1)^3=x^3-3x^2+3x-1$ so that $\dfrac{(x-1)^3+1}{x}=\dfrac{x^3-3x^2+3x-1+1}{x}=$ $\dfrac{x^3-3x^2+3x}{x}=x^2-3x+3$. As $x\to 0$, this $\to 3$

Answer to 8 3