**1A**

(12) 1. Compute the derivatives of the functions shown. In this problem, you may write only the answers and get full credit. *Please* do not "simplify" the answers!

a) $3 + 5x^3 - 12x^7$

b) $\dfrac{e^x + 7}{x^2 + 1}$

c) $\left(\dfrac{7}{x^2} + 3\cos x\right)(5x^2 + \sin x)$

(5) 2. Compute $F(2)$ and $F'(2)$ if $F(x) = x^3 f(x)$ and $f$ is differentiable with $f(2) = -3$ and $f'(2) = -1$. Show supporting work for your answers.

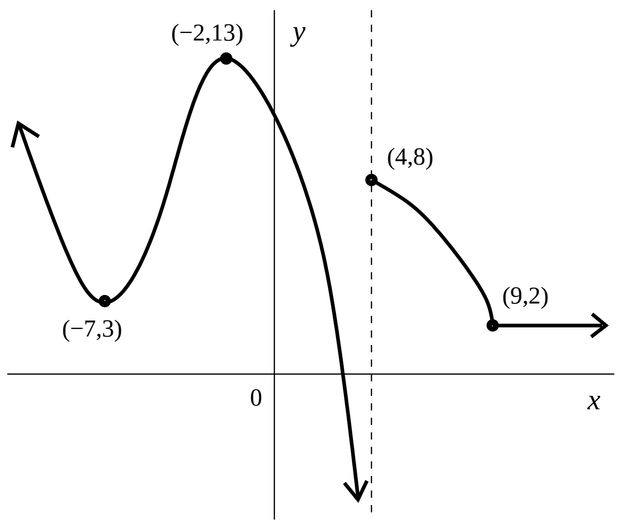
$F(2) =$ ________

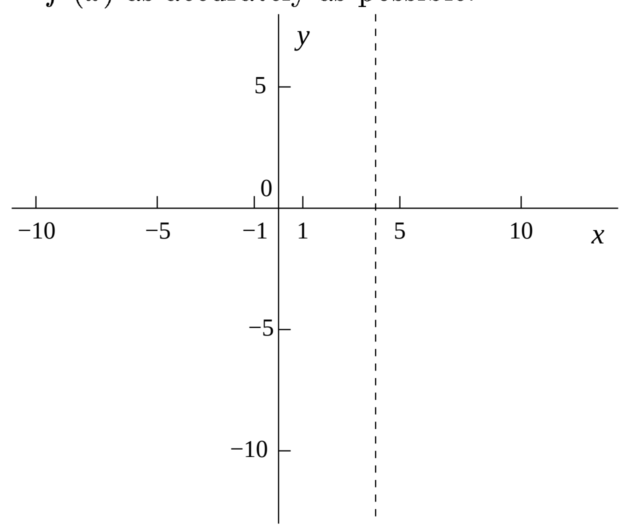
$F'(2) =$ ________

(15) 3. a) State the formal definition of the derivative, $f'(x)$, of the function $f(x)$.
b) Use your answer to a) combined with algebraic manipulation and standard properties of limits to compute the derivative of $f(x) = \dfrac{x}{x^2 + 1}$.
**Comment** For the "experts": do **NOT** use l'Hospital's rule.

(14) 4. Here is a graph of $y = f(x)$. [BELOW]

a) Use these axes to sketch a graph of $y = f'(x)$ as accurately as possible. [BELOW]




b) For which $x$'s can you conclude that $f$ is *not* continuous?

$x =$ ________ .

c) For which $x$'s can you conclude that $f$ is *not* differentiable?

$x =$ ________ and $x =$ ________ .

(16) 5. Compute these limits. Supporting work for each answer *must* be given to earn full credit.
**Comment** For the "experts": do **NOT** use l'Hospital's rule.

a) $\displaystyle\lim_{x\to 3} \frac{x - 3}{x^2 - 5x + 6}$

b) $\displaystyle\lim_{x\to +\infty} \frac{x^3 - 9x + 2}{-2x^3 + 4x^2}$

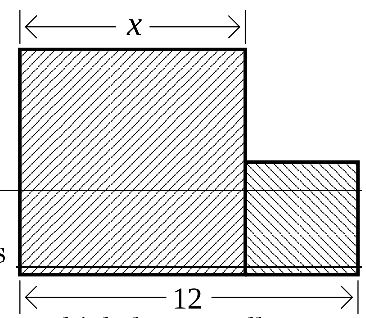
c) $\displaystyle\lim_{x\to 0} \frac{4x^2 - 3x + 2}{5e^{6x}}$

d) $\displaystyle\lim_{x\to 10^-} \frac{x}{100 - x^2}$.

(8) 6. a) Find numbers $K$ and $L$ so that $K \leq 2 + 5\sin(x^2 + 4x) \leq L$ for all numbers $x$. Give evidence to support your answer.
b) Suppose $f(x) = x^3 + 2 + 5\sin(x^2 + 4x)$. What is the sign of $f(2)$? What is the sign of $f(-2)$? You may use your answers to a) to explain your answers here.
c) Explain why the equation $f(x) = x^3 + 2 + 5\sin(x^2 + 4x) = 0$ must have at least one solution. Give an interval in which a solution must be found. You must quote a specific result from this course and explain its relevance. Your answers to b) may be useful here.

(16) 7. Suppose $f(x) = Ax + \dfrac{B}{x^2}$ where $A$ and $B$ are constants. Find values of $A$ and $B$ so that $y = 6x + 5$ is tangent to $y = f(x)$ when $x = -1$.
**Hint** Get information about $f(-1)$ and $f'(-1)$ from the given tangent line and then use this to find $A$ and $B$.

(14) 8. Two squares are placed so that their sides are touching, as shown. The sum of the lengths of one side of each square is 12 feet, as shown. Suppose the length of one side of one square is $x$ feet.
a) Write a formula for $f(x)$, the sum of the total area of the two squares. What is the domain of this function when used to describe this problem? (The domain should be related to the problem statement.)




$f(x) =$ ______

The domain of $f$ is ______

b) Suppose that $g(x)$ is the sum of twice the area of the square which has smaller area plus the area of the square which has larger area. Describe the function $g(x)$ algebraically.
**Hint** Read the question *carefully.* The answer is a piecewise-defined function. A complete answer should give all relevant information.

$$g(x) = \begin{cases} \text{\_\_\_\_\_\_\_\_} \\ \text{\_\_\_\_\_\_\_\_} \end{cases}$$

A A

# First Exam for Math 151

## Sections 4, 5, and 6

October 16, 2006

**NAME** ______________________________________

**SECTION** __________

**Do all problems, in any order.**
**Show your work. An answer alone may not receive full credit.**
**No texts, notes, or calculators may be used on this exam.**

| Problem Number | Possible Points | Points Earned: |
|---|---|---|
| 1 | 12 | |
| 2 | 5 | |
| 3 | 15 | |
| 4 | 14 | |
| 5 | 16 | |
| 6 | 8 | |
| 7 | 16 | |
| 8 | 14 | |
| Total Points Earned: | | |

A A