(12)　1. In this problem, $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$.
Find the absolute minimum and absolute maximum values of f in the interval $[-2, 1]$.

(12)　2. The program `Maple` displays the image shown to the right when asked to graph the equation

$$y^2 = x^3 - 3xy + 3.$$

a) Verify by substitution that the point $P = (-2, 1)$ is on the graph of the equation.

b) Find $\dfrac{dy}{dx}$ in terms of y and x.

c) Find an equation for the line tangent to the graph at the point $P = (-2, 1)$. You do **not** need to "simplify" your answer!

d) Sketch this tangent line in the appropriate place on the image displayed.
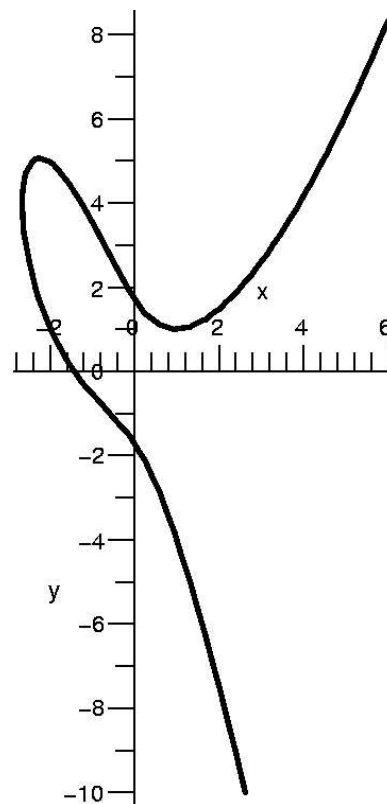
(12)　3. Ant **A** is crawling up a vertical pole at .3 meters/minute. At the same time ant **B** is crawling away from the base of the pole on the horizontal ground at .4 meters/minute.

a) If θ is the angle that ant **B** sees between the base of the pole and ant **A**, if a is the distance from ant **A** to the base of the pole, and if b is the distance from ant **B** to the base of the pole, then <u>write a formula for θ as a function of a and b</u>.

b) Use the information provided to compute θ and $\dfrac{d\theta}{dt}$ at the instant that ant **A** is 10 meters up the pole and ant **B** is 5 meters from the base of the pole. You do **not** need to "simplify" your answer!

(12)　4. Suppose $B(x)$ is a differentiable function with $\underline{B(2) = 3}$ and that the <u>derivative</u> of B is given by the following formula:

$$B'(x) = \sqrt{23 - 7x}.$$

Suppose also that $C(x) = 5x^2 - 3$. Let $A(x) = B(C(x))$.

a) Compute $A(1)$. Write a formula for $A'(x)$ only in terms of x and then compute $A'(1)$.

b) Use your answers to a) and linear approximation to find an approximate value of $A(0.95)$. You do **not** need to "simplify" your answer!

c) It is true that $A''(1) = -\dfrac{260}{3}$. Is the estimate you found in a) likely to be greater than or less than the true value of $A(0.95)$? Give reasoning which supports your answer.

(12)   5. In the right triangle $\triangle ABC$, the right angle is at C and the legs are $|AC| = 4$ and $|BC| = 12$. A rectangle is placed inside the triangle, with one corner at C and the opposite corner on the hypotenuse. What are the dimensions and area of the rectangle which has largest area? Briefly explain why you found the rectangle with <u>largest</u> area.

(12)   6. Find the limits.   a) $\lim\limits_{x \to 1} \dfrac{x^4 - 4x + 3}{(x-1)^2}$.   b) $\lim\limits_{x \to \infty} (5 + 3x)^{2/x}$.   c) $\lim\limits_{x \to \infty} \dfrac{\frac{\pi}{2} - \arctan x}{e^x - 1}$.

(22)   7. In this problem, $f(x) = \dfrac{x+1}{x^2+3}$.

a) What are $\lim\limits_{x \to +\infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$? $\lim\limits_{x \to +\infty} f(x) = $ _____ and $\lim\limits_{x \to -\infty} f(x) = $ _____ .

b) Compute $f'(x)$ *carefully*, since the result is needed for successful completion of the remainder of the problem. Simplify your result. $f'(x) = $ _____ .

c) Find all solutions of $f'(x) = 0$. For each such x, compute $f(x)$.
$x = $ _____ and $f($____$) = $ _____ . $x = $ _____ and $f($____$) = $ _____ .

d) Where is $f'(x) > 0$? Where is $f'(x) < 0$? $f'(x) > 0$ when _____ .
$f'(x) < 0$ when _____ and when _____ .

e) Sketch a graph of $y = f(x)$. The conclusions of parts a) and c) and d) should all be used here. The scales of the vertical and horizontal axes are very different.
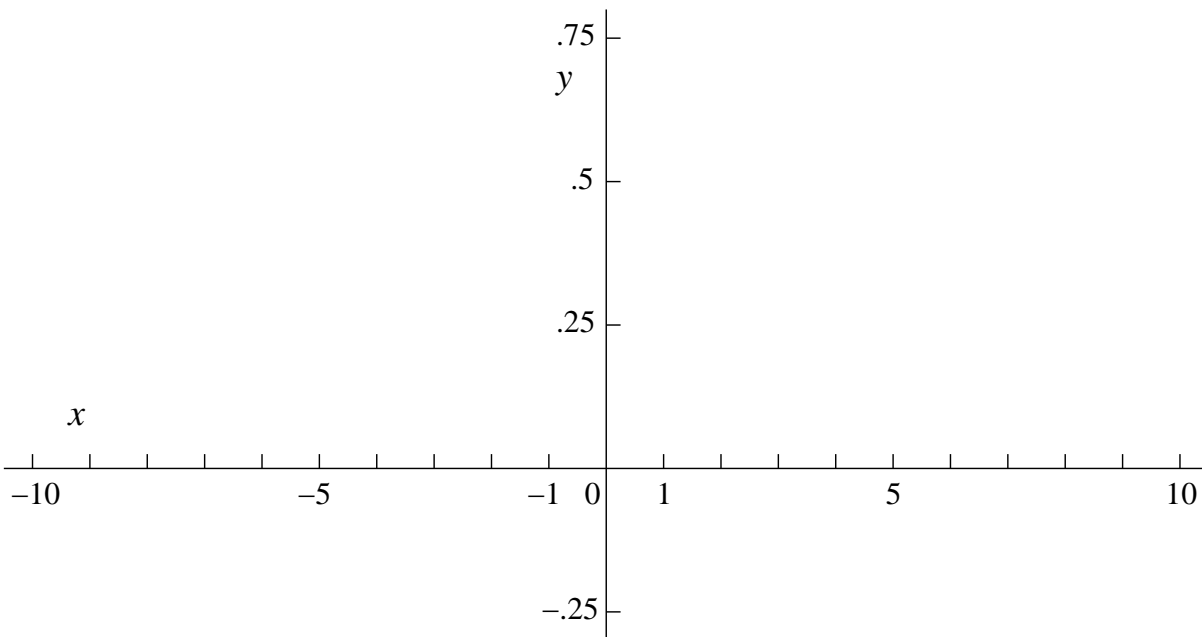
f$_1$) How many solutions does the equation $f(x) = .07$ have? (You are *not* asked to find the solutions!) $f(x) = .07$ has _____ solutions.

f$_2$) How many solutions does the equation $f(x) = .87$ have? (You are *not* asked to find the solutions!) $f(x) = .87$ has _____ solutions.

g) What is the *range* of f? (That is, the collection of all y's for which $f(x) = y$ has at least one solution.)   The range of f is _____ .

This problem continues on the next page.

h) You do <u>not</u> need to compute $f''(x)$ to answer the following question: how many inflection points must $y = f(x)$ have, and what can you say about the approximate location of these inflection points?

    The number of inflection points is _____ . Show each of the approximate values of their first coordinates on the number line below with a • .

x
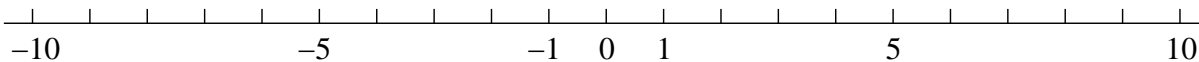
(6)   8. Suppose that $f(x)$ is a differentiable function, and that for all x, $4 < f'(x) < 6$. Suppose also that $f(0) = -2$.

a) Explain why $f(5)$ must be positive. You should quote a specific result from this course and explain its relevance.

b) Explain why $f(x) = 0$ must have a solution in the interval $[0, 5]$. You may assume the sign information stated in a) here. You should quote a specific result from this course and explain its relevance.

Second Exam for Math 151

Section 4 or 5 or 6

November 21, 2006

NAME _____

SECTION _____

Do all problems, in any order.

Show your work. An answer alone may not receive full credit.

**No texts, notes, or calculators other than the
formula sheet supplied may be used on this exam.**

Problem Number	Possible Points	Points Earned:
1	12	
2	12	
3	12	
4	12	
5	12	
6	12	
7	22	
8	6	
Total Points Earned:		