Math 151:4,5,6 **Answers to the First Computational Test (Limits)** 9/29/2006

SHOW DETAILS (algebra, limit laws) in the space next to each problem. For the experts: do **NOT** use l'Hospital's rule.

YOU MUST GIVE SOME SUPPORTING EVIDENCE. AN ANSWER ALONE WILL NOT RECEIVE FULL CREDIT.

1. $\lim_{x\to 2} \dfrac{\frac{1}{x}-\frac{1}{2}}{x-2}$

Solution $\lim_{x\to 2} \dfrac{\frac{1}{x}-\frac{1}{2}}{x-2} = \lim_{x\to 2} \dfrac{\frac{2}{2x}-\frac{x}{2x}}{x-2} = \lim_{x\to 2} \dfrac{\frac{2-x}{2x}}{x-2} = \lim_{x\to 2} \dfrac{-1}{2x} = -\dfrac{1}{4}.$

2. $\lim_{x\to 9} \dfrac{x-9}{\sqrt{x}+3}$

Solution Let $f(x) = \dfrac{x-9}{\sqrt{x}+3}$. Then $\sqrt{9}+3 \neq 0$ so $f(x)$ is continuous at 9 and $\lim_{x\to 9} f(x) = f(9) = 0$.

3. $\lim_{x\to\infty} \dfrac{6x^3+5x-8}{2x^3+9}$

Solution $\lim_{x\to\infty} \dfrac{6x^3+5x-8}{2x^3+9} = \lim_{x\to\infty} \dfrac{6+\frac{5}{x^2}-\frac{8}{x^3}}{2+\frac{9}{x^3}}$. Since $\lim_{x\to\infty} \frac{5}{x^2} = 0$ and $\lim_{x\to\infty} -\frac{8}{x^3} = 0$, $\lim_{x\to\infty} 6+\frac{5}{x^2}-\frac{8}{x^3} = 6$. Also, $\lim_{x\to\infty} \frac{9}{x^3} = 0$, so $\lim_{x\to\infty} 2+\frac{9}{x^3} = 2$. The final result is $\lim_{x\to\infty} \dfrac{6+\frac{5}{x^2}-\frac{8}{x^3}}{2+\frac{9}{x^3}} = \dfrac{6}{2} = 3$.

4. $\lim_{x\to 9^+} \dfrac{x(x-9)}{\sqrt{x}-3}$

Solution $\lim_{x\to 9^+} \dfrac{x(x-9)}{\sqrt{x}-3} = \lim_{x\to 9^+} \dfrac{x(\sqrt{x}-3)(\sqrt{x}+3)}{\sqrt{x}-3} = \lim_{x\to 9^+} x(\sqrt{x}+3) = 9\cdot 6 = 54.$

Another solution: $\lim_{x\to 9^+} \dfrac{x(x-9)}{\sqrt{x}-3} = \lim_{x\to 9^+} \dfrac{x(x-9)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)} = \lim_{x\to 9^+} \dfrac{x(x-9)(\sqrt{x}+3)}{x-9} = \lim_{x\to 9^+} x(\sqrt{x}+3) = 9\cdot 6 = 54.$

5. $\lim_{x\to -1^+} \dfrac{x^2(x-2)}{x+1}$

Solution We know $\lim_{x\to -1^+} x^2(x-2) = -3$. As x approaches -1 from the right, $x+1$ is a small positive number. The quotient in the limit expression is approximately $\frac{-3}{\text{small positive}}$, so the limit is $-\infty$.

6. $\lim_{x\to 4} \dfrac{(x-3)^2-(x-5)^2}{x-4}$

Solution $\lim_{x\to 4} \dfrac{(x-3)^2-(x-5)^2}{x-4} = \lim_{x\to 4} \dfrac{x^2-6x+9-(x^2-10x+25)}{x-4} = \lim_{x\to 4} \dfrac{4x-16}{x-4} = \lim_{x\to 4} \dfrac{4(x-4)}{x-4} = \lim_{x\to 4} 4 = 4.$