Math 151:4–6 October 10, 2006

Review material for the first exam for Math 151

The first exam will cover the first 9 lectures of the syllabus. The emphasis will be on **limits**, **continuity**, **differentiability**, and simple computation and uses of **derivatives**. No notes or calculators (or other electronic equipment!) may be used. For the "experts": do **NOT** use l'Hospital's rule. The coordinator of the course has prepared sample problems and answers which are linked to the course web page. These are useful for review (the course coordinator will be the primary author of the final exam), as are the assigned textbook homework problems. Below are sample problems written by the lecturer who will write the first exam.

> Please realize that an answer alone may not receive full credit. Appropriate supporting work is generally needed.

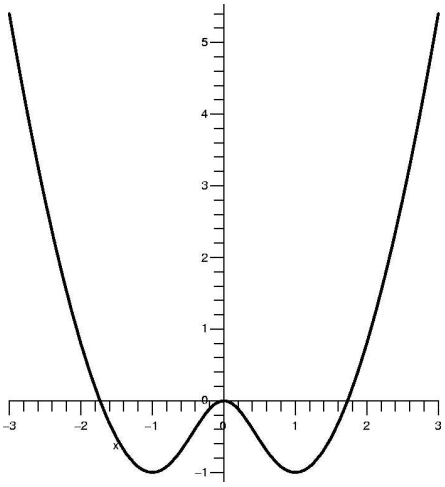
1. Suppose $f(x) = \dfrac{x^4 - 3x^2}{1 + x^2}$.

a) Compute $f'(x)$.

b) Find an equation for the line tangent to $y = \dfrac{x^4 - 3x^2}{1 + x^2}$ when $x = 2$.

c) To the right is a graph of $y = \dfrac{x^4 - 3x^2}{1 + x^2}$. Sketch the line whose equation you have found in a) on this graph.

d) For which values of x is the line tangent to $y = \dfrac{x^4 - 3x^2}{1 + x^2}$ horizontal?*



2. Suppose $f(x) = x^2 + 2 + 3\sin x$, and that g is a differentiable function about which the following is known: $g(0) = 5$ and $g'(0) = -2$. Compute the following quantities (an answer alone will *not* receive full credit): $(f + g)'(0)$; $(f \cdot g)'(0)$; $\left(\dfrac{f}{g}\right)'(0)$.

3. Write the definition of derivative as a limit and *use this definition* to find the derivative of $f(x) = \dfrac{1}{\sqrt{x}}$.

4. Suppose $f(x) = \dfrac{3}{x^2} - x^2 + e^{-7x}$. Explain why $f(x) = 0$ must have a solution in the interval $[1, 2]$. Your answer should use complete English sentences and quote a specific result from this course, explaining the relevance of this result.

OVER

---

* You *must* show work from calculus. Just writing "It looks like that" is *not* enough!.

5. Suppose that $f(x) = \dfrac{|x^2 - 1|}{x - 1}$.

a) Find $\lim_{x \to 1^+} f(x)$ and $\lim_{x \to 1^-} f(x)$.

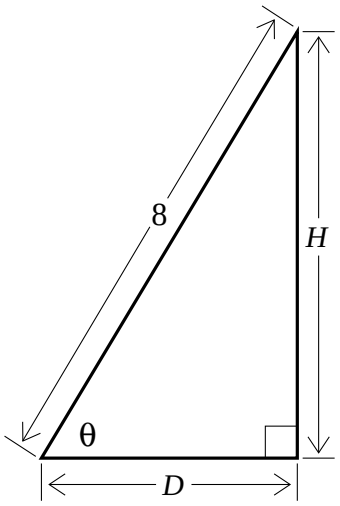
b) Does $\lim_{x \to 1} f(x)$ exist?

c) Sketch a graph of $y = f(x)$ for x between -3 and 3.*

6. a) If $f(x) = \dfrac{6e^x}{x^3 - 7}$, what is $f'(x)$? Please do not "simplify" your answer.

b) If $f(x) = (x^7 + \cos x)(4x^4 + 3x^3)$, what is $f'(x)$? Please do not "simplify" your answer.

7. A ladder which is 8 feet long has one end on flat ground and the other end on the vertical wall of a building. H is the height from the ground to the point at which the ladder touches the building, and D is the distance between the bottom of the ladder and the bottom of the wall. θ is the acute angle between the ladder and the ground.



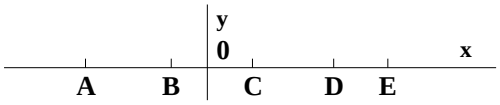
a) Write H as a function of D. That is, give a formula for H involving D and no other variable. What is the domain of this function when used to describe this problem? (The answer should be related to the problem's geometry.)

b) Write H as a function of θ. That is, give a formula for H involving θ and no other variable. What is the domain of this function when used to describe this problem? (The answer should be related to the problem's geometry.)

8. Find, as precisely as possible (you may need to use values of ln), equations for all horizontal and vertical asymptotes of $f(x) = \dfrac{5e^x + 4e^{-x}}{3e^{2x} - 2e^{-x}}$.

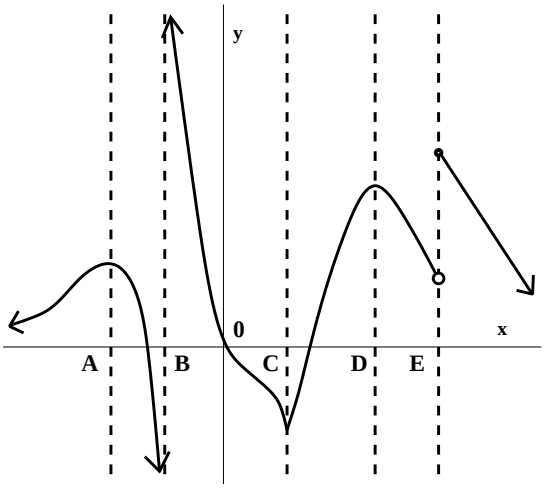
9. To the right is a graph of the function $y = f(x)$.

a) Use the axes below (make a bigger vertical axis, please!) to sketch a graph of $y = f'(x)$ as well as you can.



b) For which x's in the list {**A, B, 0, C, D, E**} is $f(x)$ continuous?

c) For which x's in the list {**A, B, 0, C, D, E**} is $f(x)$ differentiable?



* I gave this problem on a 151 exam. Those students who took that exam would agree that the graph is *tricky*.