1. An airplane is flying horizontally in a straight line and will pass directly over Newark airport. The pilot contacts a flight controller at Newark airport and is told that the line of "sight" between the airport and the plane makes an angle of 18 degrees with the horizon. The plane continues on course for 6 more miles, not yet passing over the airport, and is then told by the controller that the angle has changed to 32 degrees.

a) Make a sketch showing all the information given.

b) How high is the airplane flying, and what was the straight-line distance from the plane to the airport when the plane initially contacted the controller? Give calculations to explain your answer.

2. A cylinder is inscribed inside a sphere of radius R (an inscribed cylinder is shown in the picture to the right). Suppose the height of the cylinder is x. What is the volume of the cylinder as a function of x? (Your formula will also include R in some way.) Please be sure to give the domain of this function with some explanation.
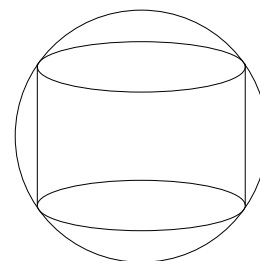
Comments You must *label* the picture and analyze it carefully.

3. A bug named Fred crawls up the y-axis at constant speed so that its (his?) position at time t (t here is a positive number) is $(0, t)$. Also another bug named Jane (!) crawls on the curve $y = \sqrt{x}$ so that the first coordinate of Jane's position at time t is t.

a) Draw a picture showing the bugs at time $t = 4$. Also draw the line segment connecting the positions of the bugs at that time. What is the slope and length of this line segment?

b) Draw a picture showing the bugs at time $t = 100$. Also draw the line segment connecting the positions of the bugs at that time. What is the slope and length of this line segment?

c) What happens when t gets very large positive to the length of the line segment, and what happens to the slope of the line segment? Explain your statements as well as you can. Use appropriate drawing and algebraic analysis of the situation.

4. a) The function S (the "squaring function") has domain all of $\mathbb{R}$ (all real numbers) and its values are given by $S(x) = x^2$ for all x. Now consider the function T whose domain is also all of $\mathbb{R}$ and which is defined by

$$T(x) = \begin{cases} S(x) & \text{if } x \neq 3 \\ 7 & \text{if } x = 3 \end{cases}$$

Sketch a graph of T. What is $\lim_{x \to 5} T(x)$? What is $\lim_{x \to 3} T(x)$? Support your assertions.

b) Suppose S is again the squaring function defined above. Now an evil interstellar visitor comes and *changes* exactly *one million values* of S and thus creates a new function, V. What can you say about $\lim_{x \to a} V(x)$ for all values of a? Support your assertions.

Why these particular problems at this particular time?

Some answers to this question are linked to the course web page.