1. a) The function S (the "squaring function") has domain all of $\mathbb{R}$ (all real numbers) and its values are given by $S(x) = x^2$ for all x. Now consider the function T whose domain is also all of $\mathbb{R}$ and which is defined by

$$T(x) = \begin{cases} S(x) & \text{if } x \neq 3 \\ 7 & \text{if } x = 3 \end{cases}$$

Sketch a graph of T. What is $\lim_{x\to 5} T(x)$? What is $\lim_{x\to 3} T(x)$? Support your assertions.

b) Suppose S is again the squaring function defined above. Now an evil interstellar visitor comes and *changes* exactly *one million values* of S and thus creates a new function, V. What can you say about $\lim_{x\to a} V(x)$ for all values of a? Support your assertions.

2. For each of the four cases below, sketch a graph of a function that satisfies the stated conditions. In each case, the *domain* of the function should be *all real numbers*.

a) $\lim_{x\to 2} f(x) = 3$ and $f(2) = 4$.

b) $\lim_{x\to 0} f(x)$ does not exist, and $|f(x)| < 2$ for all x.

c) $\lim_{x\to 1} f(x)$ exists and its value is $f(1) + 2$.

d) $\lim_{x\to -1^-} f(x)$ and $\lim_{x\to -1^+} f(x)$ do not exist, $|f(x)| < 3$ for all x, and $f(-1) = -2$.

3. Let $f(x) = x^3 + 5x - 1$ and consider the equation $f(x) = 10$.

a) Prove *without the help of a graph* that the equation $f(x) = 10$ has a solution x in the interval $(0, 2)$.

You can look at a graph, but your reasoning must use the Intermediate Value Theorem without use of a graph.

b) Use the Intermediate Value Theorem again to decide whether the solution in a) lies in $[0, 1]$ or in $[1, 2]$.

c) Continue "zooming in on" the solution, deciding at each step whether the solution lies in the left or right half of the latest interval. Stop when you are able to to state the solution with an uncertainty of no more than .05. State it!

This is the "method of bisection". You could of course find the solution by zooming in on the graph with your calculator, or by making tables with your calculator with smaller and smaller values of ΔTbl. The advantage of the method of bisection is that it is very efficient in the sense that it doesn't have to compute too many values of $f(x)$. How many values of $f(x)$ did you have to calculate?

4. $f(x)$ is a piecewise function defined as follows:

$$f(x) = \begin{cases} 2x^2 + 2, & \text{if } x < 1 \\ ax^2 + bx, & \text{if } 1 \leq x \leq 2 \\ 10 - \dfrac{2}{x}, & \text{if } x > 2 \end{cases}.$$

a) Suppose that $a = 2$ and $b = -3$. Graph $f(x)$ for $0 \leq x \leq 3$. Find the left and right hand limits of $f(x)$ as x approaches 1 and as x approaches 2.

b) Find a and b so that the graph of $f(x)$ doesn't have any jumps. Graph the resulting function $f(x)$ for $0 \leq x \leq 3$.