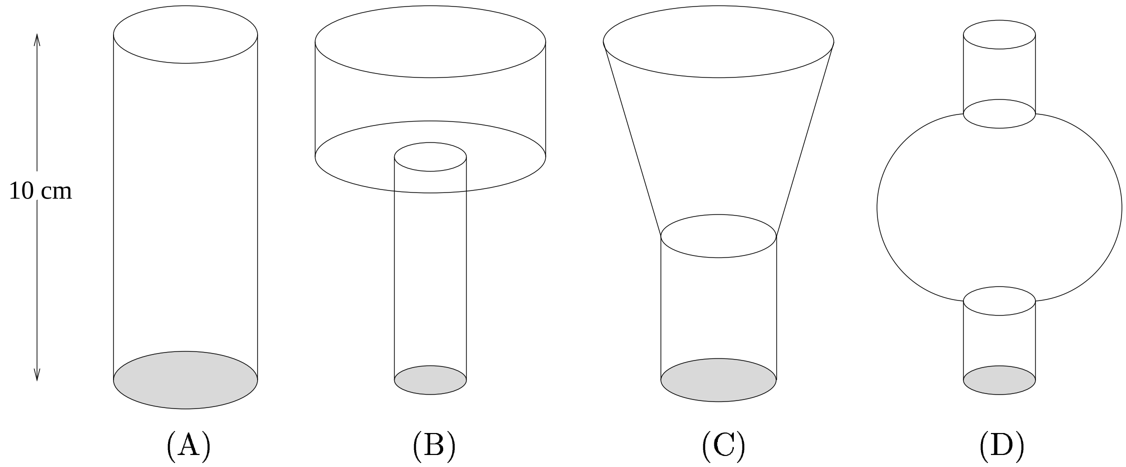
1. Four containers are each 10 cm tall. Each of them has a volume of 30 cm^3 and are each being filled by a liquid at the rate of 5 cm^3 per minute. Here is a picture of the containers:



a) For each of the containers, graph the height, $h(t)$, of the level of the liquid in the containers measured in centimeters as a function of time, t, measured in minutes.

b) Which of the functions graphed in a) are continuous? Explain your answers.

c) Which of the functions graphed in a) are differentiable? Explain your answers.

2. a) Show from the definition of derivative that

i) $f(x) = x|x|$ is differentiable at $x = 0$.
ii) $g(x) = (1 + |x|)^{1/2}$ is *not* differentiable at $x = 0$.

Hint Consider limits from "both sides" separately: $x \to 0^-$ and $x \to 0^+$. Then use the definition of $|\ |$ to complete your analysis.

b) Graph each function carefully near $x = 0$ and discuss how the graphs appear to confirm the results of a).

3. Questions about asymptotic growth "near" ∞ occur naturally when computer scientists analyze algorithms. One seemingly simple problem is sorting. How many comparisons are required to sort a list of n numbers? One reference* gives the following average running times for several sorting algorithms as a function of n:

Name	**Running time**
Comparison	$4n^2 + 10n$
Merge exchange	$3.7n(\ln n)^2$
Heapsort	$23.08n \ln n + 0.2n$

Which sorting method would you rather use if, in your application, $10 \le n \le 20$ (*e.g.*, sorting a bridge hand)? Which would you rather use if, in your application, $100 \le n \le 150$ (*e.g.*, sorting grades in a lecture course)? Which would you rather use if $n \approx 10^6$ (*e.g.*, sorting license plate numbers in New Jersey)? What happens to these functions as $n \to \infty$? Later we will be more precise about methods to compare rates of growth.

* *Sorting and Searching*, volume 3 of *The Art of Computer Programming*, by Donald Knuth.