1. Suppose f is a differentiable function whose domain is all of $\mathbb{R}$. Also suppose f has the following properties:

- $\lim_{x\to-\infty} f(x) = -2$ and $\lim_{x\to+\infty} f(x) = +\infty$.
- $f'(x) = 0$ only at -3, 0, and 2.
- $f'(x) > 0$ only for x in these intervals: $(-\infty, -3)$ and $(2, +\infty)$.
- $f(x) = 0$ only at -4, 0, and 5.
- $f(-3) = 3$ and $f(2) = -4$.

a) Sketch the simplest graph you can of $y = f(x)$ consistent with the information above.

b) Explicitly identify (use equations and complete English sentences as necessary) any vertical or horizontal asymptotes of $f(x)$, and any local or global maxima or minima of $f(x)$. Also identify any intervals where $f(x)$ is increasing or decreasing.

c) Sketch the simplest graph you can of $y = f'(x)$ consistent with the information in this problem.

d) Identify as clearly as you can any points of inflection and intervals where $f(x)$ is concave up and concave down. If you cannot locate the inflection points explicitly, be as clear as you can about the location of each inflection point.

e) Sketch the simplest graph you can of $y = f''(x)$ consistent with the information in this problem.

2. Suppose $f(x) = \dfrac{e^x - 5}{e^{2x} - 9}$.

a) What is the domain of $f(x)$? Find any vertical or horizontal asymptotes of $y = f(x)$. Give the equations for these asymptotes and show supporting reasoning.

b) Compute $f'(x)$ and simplify the result sufficiently so that you can find any critical numbers of $f(x)$. What is the sign of $f'(x)$ as $x \to +\infty$? What is the sign of $f'(x)$ as $x \to -\infty$?

c) Use the information obtained in this problem to describe the range of $f(x)$: that is, explicitly describe as well as you can the collection of y's for which $f(x) = y$ has a solution. You must show supporting reasoning for your answer.

3. Suppose $f(x) = \dfrac{1 + 2x}{2 + x^2}$.

a) What is the domain of $f(x)$? Find any vertical or horizontal asymptotes of $y = f(x)$. Give the equations for these asymptotes and show supporting reasoning.

b) Compute $f'(x)$ and simplify the result sufficiently so that you can find any critical numbers of $f(x)$. What is the sign of $f'(x)$ as $x \to +\infty$? What is the sign of $f'(x)$ as $x \to -\infty$?

c) Use the information obtained in this problem to describe the range of $f(x)$: that is, explicitly describe as well as you can the collection of y's for which $f(x) = y$ has a solution. You must show supporting reasoning for your answer.

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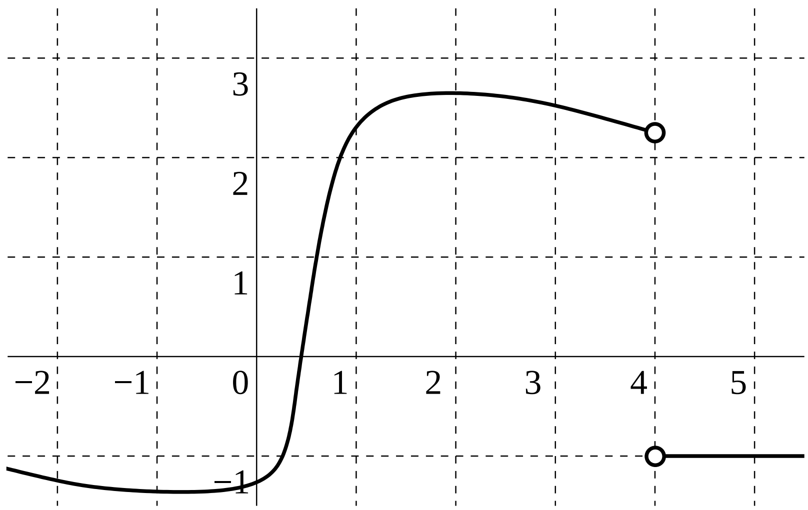
4. A graph of the derivative of $f(x)$ follows. Information about the function $f(x)$ is known only for $-2.5 < x < 4.5$. You are told:

- $f(x)$ is continuous everywhere in that interval.
- $f(x)$ is differentiable everywhere in that interval except at one value of x.
- Also $\underline{f(-2) = 1}$.

You will need this information in addition to the graph of $f'(x)$ to answer the questions which follow. Please look at the graph carefully, and consider the information in both the numbers and the shapes of the graph (both "quantitative" and "qualitative" information)!



A graph of $y = f'(x)$, the derivative of $f(x)$

a) Explain why $-2 < f(0) < -1$. You must look carefully at the graph and make estimates using the MVT. Explain the steps of your reasoning in detail.

b) Explain why $f(3) > 4 + f(1)$. Again, use the MVT and explain your reasoning in detail.

c) What can you say about $f(1) - f(0)$? How big and how small can this number be?

d) Use the information in a), b), and c) to explain why $f(3)$ must be positive.

e) Explain why the equation $f(x) = 0$ must have a solution between 0 and 3. You will need the IVT and the information obtained in previous parts of this problem.

f) Sketch a graph of $y = f(x)$ as well as you can using the information present.

5. Compute $\lim_{x \to +\infty} x^{1/x}$ and $\lim_{x \to 0^+} x^{1/x}$.