(12) 1. The program `Maple` displays the image shown to the right when graphing the equation

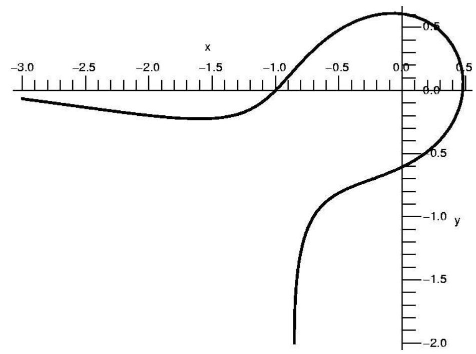
$$\ln(x^2+y^2)-xy+x=-1.$$

This problem has **three parts**.

a) Verify by substitution in the equation that the point $(-1,0)$ is on the graph of the equation.

b) Use calculus to find an equation for the line tangent to the graph at the point $(-1,0)$.

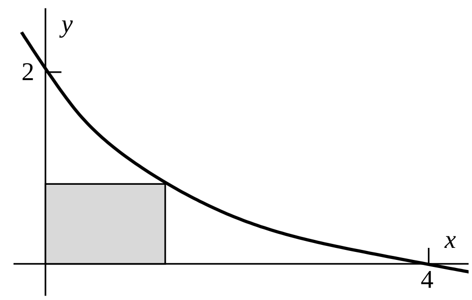
c) Sketch this tangent line in the appropriate place on the image displayed.



(12) 2. Find the area of the largest rectangle that can be inscribed in the region bounded by the graph of $y=\dfrac{4-x}{2+x}$ and the coordinate axes.

Comment A correct graph is shown to the right. You do *not* need to "simplify" your answer, but you must give some justification that you have found the rectangle with *largest* area.

The largest area is ________________.



(12) 3. At a certain time, a rectangle has **L**ength equal to 5 inches and **W**idth equal to 7 inches. Also at that time, the **L**ength is increasing at .3 inches per second and the **W**idth is decreasing at .4 inches per second.

a) Compute the area of the rectangle at the certain time. Is the area increasing or decreasing at that time, and at what rate? Put your answers in the spaces indicated.

Area = _______ inches^2

Rate of change of area = _______ inches^2 per second

The area is ____creasing (use one of {in|de}).

b) Compute the length of the diagonal of the rectangle at the certain time. Is this length increasing or decreasing at that time, and at what rate? Put your answers in the spaces indicated.

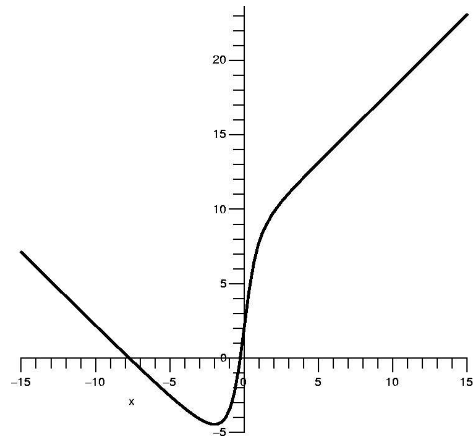
Diagonal's length = _______ inches

Rate of change of diagonal's length = __________ inches per second

The diagonal's length is ____creasing (use one of {in|de}).

(20) 4. Here $f(x) = \dfrac{x^2+8x+2}{\sqrt{x^2+1}}$. A `Maple` graph of $y = f(x)$ is shown to the right. These equations are true:

$$f'(x) = \frac{x^3+8}{(x^2+1)^{3/2}} \text{ and } f''(x) = \frac{3x(x-8)}{(x^2+1)^{5/2}}.$$



a) Compute $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$. Give evidence (not just the graph!) supporting your assertions.

b) Where is f increasing? Where is f decreasing? Support your assertions using results from the course and information supplied.

c) Exactly how many roots does the equation $f(x) = 100$ have? Do *not* try to compute these roots but use results from the course and your answers to a) and b) to verify your answer.

d) Where is the graph of $y = f(x)$ concave up and concave down? Label any points of inflection on the graph. (You need not give support for your answer here but indicate intervals clearly below, and label the graph as needed.)

(16) 5. Suppose $f'(x) = x(x-1)^2(x-2)^3(x-3)$.

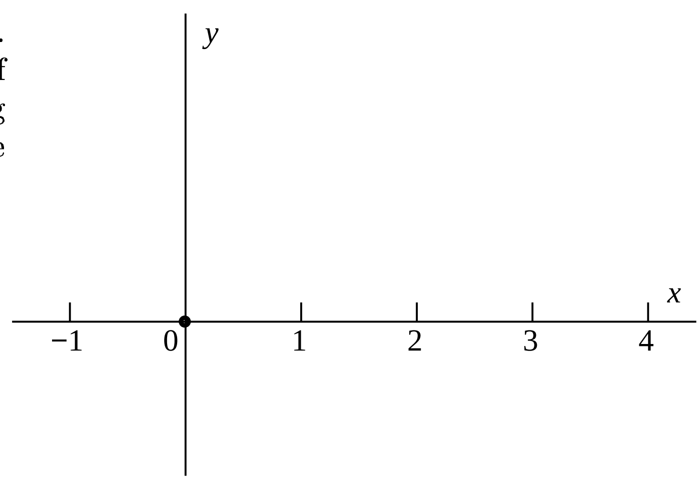
Comment and advice This is a formula for the derivative of f. Please do *not* "expand" or "simplify" the formula, and do *not* try to find f explicitly.

a) Where are the critical points of f and what are their types? Enter the information in the table below. The types should be selected from the words MAXIMUM, MINIMUM, and NEITHER. *Briefly* give reasoning which supports your assertions about this function in the space after the table.

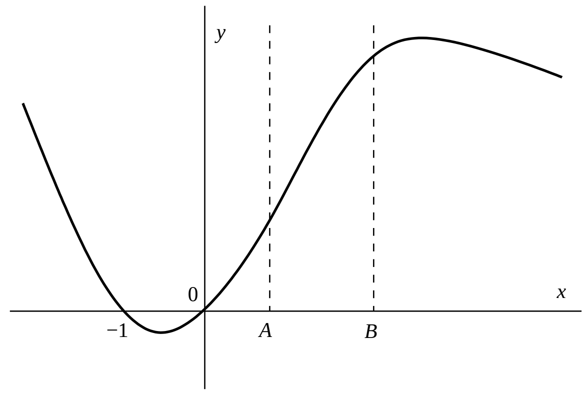
Critical point: $x =$				
Type of critical point				

Explanation

b) Suppose you know that $f(0) = 0$. Sketch a qualitatively correct graph of $y = f(x)$ on the interval $[-1, 4]$ using the axes below. To help, a point on the graph at $(0, 0)$ has already been drawn.



(10) 6. A graph of $y = f(x)$ is shown below. The equation $f(x) = 0$ has solutions when $x = 0$ and when $x = -1$.



a) Suppose that Newton's method is applied to find a solution of $f(x) = 0$ with starting value $x = A$. Show on the graph's x–axis as well as you can (with labels!) the result of the first two iterations A_1 and A_2 of Newton's method. What is the limit of the sequence of iterations $\{A_n\}$ that begins with $x = A$?

Draw appropriate supporting evidence for your A_1 and A_2 on the graph. Your limit assertion needs no supporting evidence.

$$\lim_{n\to\infty} A_n = \_\_\_\_\_\_$$

b) Suppose that Newton's method is applied to find a solution of $f(x) = 0$ with starting value $x = B$. Show on the graph's x–axis as well as you can (with labels!) the result of the first two iterations B_1 and B_2 of Newton's method. What is the limit of the sequence of iterations $\{B_n\}$ that begins with $x = B$?

Draw appropriate supporting evidence for your B_1 and B_2 on the graph. Your limit assertion needs no supporting evidence.

$$\lim_{n\to\infty} B_n = \_\_\_\_\_\_$$

(10) 7. Compute these limits. Supporting work for each answer *must* be given to earn full credit.

Comment Your results should be a specific number, $+\infty$, $-\infty$, or Does Not Exist.

a) $\displaystyle\lim_{x\to 0} \frac{e^{(5x^2)} - 1}{x^2 - x^3}$

b) $\displaystyle\lim_{x\to 0} (1 + k\sin x)^{1/x}$ (here k is a positive constant)

c) $\displaystyle\lim_{x\to 5} \frac{x^2 - 25}{x + 5}$.

(8) 8. A car moves along a road from the town of FROG to the town of TOAD. Its speed is always between 40 and 60 mph. The car leaves FROG at 9 AM and enters TOAD at 10:30 AM.

Write a mathematical model of this situation. Your model should include a function with a domain you define, with properties of the function stated carefully. Use this model together with *specific results* from Math 151 to obtain some estimate of the distance along the road from FROG to TOAD.

A A

Second Exam for Math 151

Sections 7, 8, and 9

November 16, 2007

NAME ____

SECTION __________

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No texts, notes, or calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	12	
3	12	
4	20	
5	16	
6	10	
7	10	
8	8	
Total Points Earned:		

A A