1(a) In this problem, $f(x) = x^{1/3}$ and $f'(x) = (1/3)x^{-2/3}$. Thus, $f(8) = 2$ and $f'(8) = 1/12$. Then, $L(x) = f(8) + f'(8)(x-8) = 2 + (x-8)/12$.

1(b) If $x = 7$, $7^{1/3} = f(7) \approx L(7) = 2 - 1/12 = 23/12$. If $x = 9$, $9^{1/3} \approx L(9) = 2 + 1/12 = 25/12$.

2. With $f(x) = 2\sin(x) + \cos(2x)$, then $f'(x) = 2\cos(x) - 2\sin(2x)$. Using $\sin(2x) = 2\sin x \cos x$, $f'(x) = 2\cos x - 4\sin x\cos x$, or $f'(x) = 2\cos x(1 - 2\sin x)$.

2(a) If $f'(x) = 0$, then either $\cos x = 0$ or $1 - 2\sin x = 0$. If $\cos x = 0$, since $0 \le x \le 2\pi$, either $x = \pi/2$ or $x = 3\pi/2$. If $1 - 2\sin x = 0$, then $\sin x = 1/2$. Since $0 \le x \le 2\pi$, $x = \pi/6$ or $x = 5\pi/6$. Thus, the critical points are $x = \pi/6, \pi/2, 5\pi/6, 3\pi/2$.

2(b) The maximum and minimum of f on the interval $[0, 2\pi]$ occur at either the endpoints $x = 0$ and $x = 2\pi$ or at one of the critical points of (a). The above table gives the value of f at each of these points. It follows that the maximum value is $3/2$ and the minimum value is -3.

x	0	$\pi/6$	$\pi/2$	$5\pi/6$	$3\pi/2$	2π
$f(x)$	1	$3/2$	1	$3/2$	-3	1

3(a) Since the limit is of type $0/0$, L'Hospital's Rule is applicable.

Then, $\displaystyle\lim_{x\to 1} \frac{x^k - kx + k - 1}{(x-1)^2} = \lim_{x\to 1} \frac{kx^{k-1} - k}{2(x-1)} = \lim_{x\to 1} \frac{k(k-1)x^{k-2}}{2} = \frac{k(k-1)}{2}$.

3(b) Here the limit is of type 1^∞. Thus, after first taking ln( ), L'Hospital's Rule is used.

Let $y = (1+kx)^{1/x}$. Then, $\ln y = \ln(1+kx)^{1/x} = \dfrac{\ln(1+kx)}{x}$. Thus, $\displaystyle\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(1+kx)}{x} = \lim_{x\to 0^+} \frac{k/(1+kx)}{1} = k$. Then, as $x \to 0^+$, $\ln y \to k$. It follows that: $y = e^{\ln y} \to e^k$.

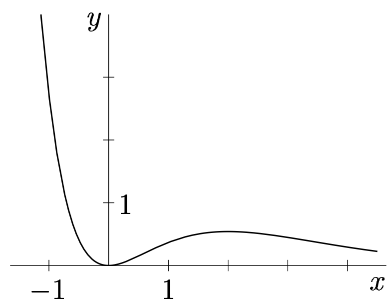
3(c) Here, $\displaystyle\lim_{x\to\infty} (\ln(2e^x + x^k) - x) = \lim_{x\to\infty} (\ln(e^x(2 + x^k/e^x) - x) = \lim_{x\to\infty} (\ln(e^x) + \ln(2 + x^k/e^x) - x) = \lim_{x\to\infty} (x + \ln(2 + x^k/e^x) - x) = \lim_{x\to\infty} \ln(2 + x^k/e^x) = \ln 2$, since as $x \to \infty$, $x^k/e^x \to 0$.

4(a) $3x^2\sin^2(2^x) + 2(\ln 2)x^3 2^x \sin(2^x)\cos(2^x)$ 4(b) $e^x/(1+e^{2x})$

4(c) $\left((\cos x)(\ln(1+x^2)) + \dfrac{2x\sin x}{1+x^2}\right)(1+x^2)^{\sin x}$

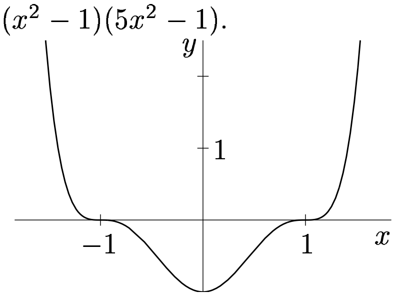
5(a) Here, $f(x) = x^2e^{-x}$, $f'(x) = (2x - x^2)e^{-x}$, and $f''(x) = (x^2 - 4x + 2)e^{-x}$. It follows that the critical points are $x = 0$ and $x = 2$. Since $e^{-x} > 0$, the sign of $f'(x)$ is that of $x(2-x)$. Thus, if $x < 0$, $f'(x) < 0$. If $0 < x < 2$, $f'(x) > 0$, and if $x > 2$, $f'(x) < 0$. Thus f is decreasing on $(-\infty, 0)$, increasing on $(0, 2)$, decreasing on $(2, \infty)$. It follows that a local minimum occurs at $x = 0$ and a local maximum at $x = 2$.

Since $\lim_{x\to\infty} x^2e^{-x} = 0$, as $x \to \infty$, $y = 0$ is a horizontal asymptote. Solving $x^2 - 4x + 2 = 0$, it follows that the inflection points are: $x = 2 \pm \sqrt{2}$. The graph appears to the right.



5(b) In this problem, $f(x) = (x^2-1)^3$, $f'(x) = 6x(x^2-1)^2$, and $f''(x) = 6(x^2-1)(5x^2-1)$. Thus, the critical points are $x = 0$ and $x = \pm 1$. Since $(x^2-1)^2 \ge 0$, the sign of $f'(x)$ is that of x. Thus, if $x < 0$, $f'(x) \le 0$ and if $x > 0$, $f'(x) \ge 0$. Thus f is decreasing on $(-\infty, 0)$, increasing on $(0, \infty)$.

For this reason, the critical points $x = \pm 1$ are neither local maxima nor local minima. Since the sign of f' changes from negative to positive at $x = 0$, a local minimum occurs at $x = 0$. The inflection points are: $x = \pm 1$, $x = \pm 1/\sqrt{5}$. The graph is at the right.



6(a) By the Mean Value Theorem, $f(x) - f(0) = f'(c)(x - 0)$ with $0 < c < x$. Since $f(0) = 10$ and $f'(c) \le -2$, $f(x) \le 10 - 2x$. Thus, $f(5) \le 0$. By the Intermediate Value Therorem, f has a root in $[0, 5]$.

(b) The smallest a is 5. By (a) f has a root in $[0, 5]$. If $f(x) = 10 - 2x$, then $f(0) = 10$ and $f'(x) = -2$. Also the root of f is exactly $x = 5$. It follows that a cannot be smaller than 5.

(c) No, Since if f had two positive roots a, b with $a < b$, by Rolle's Theorem, $f'(c) = 0$ for some c with $a < c < b$, contrary to $f'(x) \le -2$, for $x \ge 0$.

7(a) In this problem, $f(x) = x^3 + x - 1$ and $f'(x) = 3x^2 + 1$. It follows that $x - \dfrac{f(x)}{f'(x)} = x - \dfrac{x^3 + x - 1}{3x^2 + 1} = \dfrac{2x^3 + 1}{3x^2 + 1}$. Thus, the recursion is given by $x_{n+1} = \dfrac{2{x_n}^3 + 1}{3{x_n}^2 + 1}$.

7(b) If $x_0 = 1$, then $x_1 = \dfrac{2+1}{3+1} = \dfrac{3}{4}$. Then, $x_2 = \dfrac{2(3/4)^3 + 1}{3(3/4)^2 + 1} = \dfrac{2{\cdot}3^3 + 4^3}{4{\cdot}3^3 + 4^3} = \dfrac{59}{86}$.

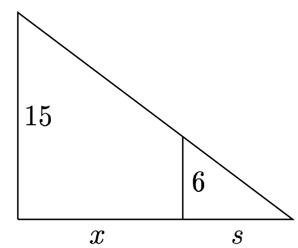
8(a) Using implicit differentiation, $\quad 2\dfrac{dy}{dx} + \cos(x+y)\left(1 + \dfrac{dy}{dx}\right) = 2.$

Thus, $(2 + \cos(x+y))\dfrac{dy}{dx} = 2 - \cos(x+y)$, or $\dfrac{dy}{dx} = \dfrac{2 - \cos(x+y)}{2 + \cos(x+y)}$.

8(b) If $x = \pi/2$, $y = \pi/2$, then $\cos(x+y) = \cos(\pi) = -1$, and $dy/dx = 3/1 = 3$. Thus, the equation of the tangent line is $y - \pi/2 = 3(x - \pi/2)$, or $y = 3x - \pi$.

8(c) Since $-1 \le \cos(x+y) \le 1$, $1 \le 2+\cos(x+y) \le 3$. Also, as $-1 \le -\cos(x+y) \le 1$, $1 \le 2-\cos(x+y) \le 3$. Since dy/dx is the quotient of two positive numbers, it follows that $dy/dx > 0$, and thus f is increasing.

9. Let x be the distance of the man from the lamppost and s the length of his shadow. (See the picture to the right.) In the picture the larger triangle is similar to the smaller triangle, and so it follows that $\dfrac{x+s}{15} = \dfrac{s}{6}$, or $6(x+s) = 15s$, or $6x = 9s$, or $2x = 3s$.

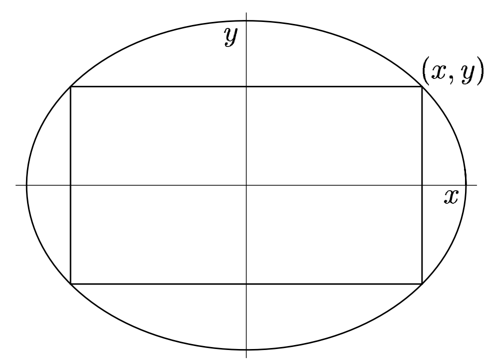


9(a) 8 ft. If the man is 12 ft. from the lamppost, $x = 12$, and so $s = 8$ ft.

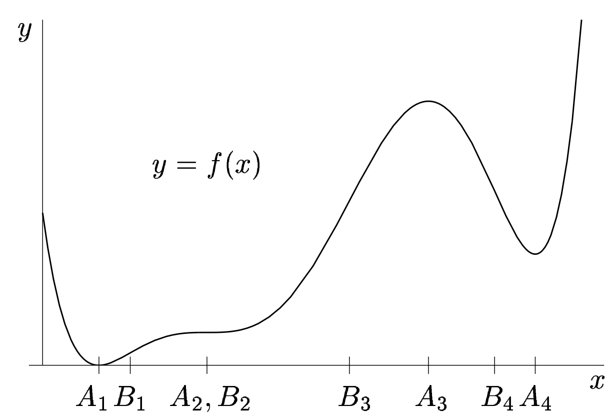
9(b) 2 ft./sec. Differentiating $2x = 3s$, $2\,dx/dt = 3\,ds/dt$. Thus, if $dx/dt = 3$, $ds/dt = 2$ ft./sec.

10. Suppose that (x, y) is the upper right hand corner of the rectangle that has been inscribed in the ellipse. Then, $x > 0, y > 0$. (See the picture at the right.) Since the width of the rectangle is $2x$ and the height is $2y$, the area of the rectangle is $4xy$. Also, (x, y) lies on the ellipse $x^2/a^2 + y^2/b^2 = 1$. Solving for y in terms of x and using $y > 0$, $y = (b/a)\sqrt{a^2 - x^2}$. It follows that $A(x) = (4b/a)x\sqrt{a^2 - x^2}$. Differentiating $A(x)$, $A'(x) = (4b/a)\sqrt{a^2 - x^2} + (4b/a)x(-1/2)(-2x/\sqrt{a^2 - x^2}) = (4b/a)(a^2 - 2x^2)/\sqrt{a^2 - x^2}$. Thus, if $A'(x) = 0$, $2x^2 = a^2$, or as $x > 0$, $x = a/\sqrt{2}$. Substituting in the formula for y, $y = b/\sqrt{2}$. Since the height and width double x and y, the dimensions of the largest rectangle are $\sqrt{2}a \times \sqrt{2}b$.

10.



11. Since $y = f'(x)$ crosses the x-axis at $x = A_1, A_2, A_3, A_4$, $f'(x) = 0$ if $x = A_1, A_2, A_3, A_4$. Thus, the critical points of $y = f(x)$ are at $x = A_1, A_2, A_3, A_4$.
Similarly, since the derivative of f' is 0 at $x = B_1, B_2, B_3, B_4$, the inflection points of $y = f(x)$ are at $x = B_1, B_2, B_3, B_4$.
The graph of $y = f(x)$ appears at the right.
The graph should show: horizontal tangents at $x = A_1, A_2, A_3, A_4$, and $f(A_1) = 0$.
In addition, the graph should show f decreasing on $(-\infty, A_1)$, increasing on (A_1, A_2) and (A_2, A_3), f decreasing on (A_3, A_4), and increasing on (A_4, ∞).



12. Integrating once, $v(t) = ds/dt = 15t^4 - 6t^2 + C$. Since $v(0) = -3$, $C = -3$, and $v(t) = 15t^4 - 6t^2 - 3$. Integrating, $s(t) = 3t^5 - 2t^3 - 3t + D$. Since $s(0) = 4$, $D = 4$. Thus, $s(t) = 3t^5 - 2t^3 - 3t + 4$.