SHOW DETAILS (algebra, limit laws) in the space next to each problem. Do **NOT** use l'Hôpital's rule. Answers may be a specific number, $+\infty$, $-\infty$, or Does Not Exist (DNE).

YOU MUST GIVE SOME SUPPORTING EVIDENCE. AN ANSWER ALONE WILL NOT RECEIVE FULL CREDIT.

1. $\lim\limits_{x\to 2} \dfrac{x^3 - 4x}{x - 2}$

Answer $\lim\limits_{x\to 2} \dfrac{x^3 - 4x}{x - 2} = \lim\limits_{x\to 2} \dfrac{x(x-2)(x+2)}{x-2} = \lim\limits_{x\to 2} x(x+2) = 8.$

2. $\lim\limits_{x\to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x + 2}$

Answer If $f(x) = \dfrac{\frac{1}{x} - \frac{1}{2}}{x + 2}$, then $2 + 2 \neq 0$ so $f(x)$ is continuous at 2 and $\lim\limits_{x\to 2} f(x) = f(2) = 0.$
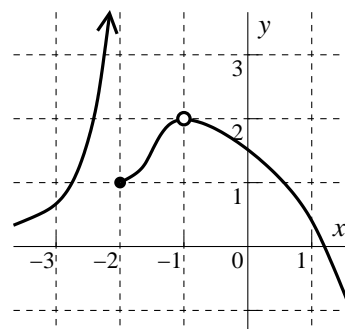
3. $\lim\limits_{x\to 1} \dfrac{x^2 - 3x + 2}{2x^2 - x - 1}$

Answer $\lim\limits_{x\to 1} \dfrac{x^2 - 3x + 2}{2x^2 - x - 1} = \lim\limits_{x\to 1} \dfrac{(x-1)(x-2)}{(2x+1)(x-1)} = \lim\limits_{x\to 0} \dfrac{x-2}{2x+1} = -\dfrac{1}{3}.$

4. $\lim\limits_{x\to 0} \dfrac{1 - \sqrt{x^2 + 1}}{x^2}$

Answer $\lim\limits_{x\to 0} \dfrac{1 - \sqrt{x^2 + 1}}{x^2} = \lim\limits_{x\to 0} \left(\dfrac{1 - \sqrt{x^2 + 1}}{x^2}\right)\left(\dfrac{1 + \sqrt{x^2 + 1}}{1 + \sqrt{x^2 + 1}}\right) = \lim\limits_{x\to 0} \dfrac{1 - (x^2 + 1)}{x^2\left(1 + \sqrt{x^2 + 1}\right)}$

$= \lim\limits_{x\to 0} \dfrac{-x^2}{x^2\left(1 + \sqrt{x^2 + 1}\right)} = \lim\limits_{x\to 0} \dfrac{-1}{1 + \sqrt{x^2 + 1}} = -\dfrac{1}{2}.$

5. Use the graph displayed to the right to complete the limit statements below. No justification needs to be given.

$y = f(x)$

Answers to 5

$\lim\limits_{x\to -2^-} f(x) = +\infty \qquad \lim\limits_{x\to -2^+} f(x) = 1 \qquad \lim\limits_{x\to -1} f(x) = 2$

6. $\lim\limits_{x\to 4} \dfrac{\frac{1}{x^2} - \frac{1}{16}}{x - 4}$

Answer $\lim\limits_{x\to 4} \dfrac{\frac{1}{x^2} - \frac{1}{16}}{x - 4} = \lim\limits_{x\to 4} \dfrac{\frac{16 - x^2}{16x^2}}{x - 4} = \lim\limits_{x\to 4} \dfrac{16 - x^2}{16x^2(x - 4)} = \lim\limits_{x\to 4} \dfrac{(4-x)(4+x)}{16x^2(x-4)} = \lim\limits_{x\to 4} \dfrac{-(4+x)}{16x^2}$

$= \dfrac{-8}{16 \cdot 4^2} = -\dfrac{1}{32}.$