1. The function S (the "squaring function") has domain all real numbers and is defined by the formula $S(x) = x^2$ for all x.

a) Consider the function T whose domain is also all real numbers which is defined by

$$T(x) = \begin{cases} S(x) & \text{if } x \neq 3 \\ 7 & \text{if } x = 3 \end{cases}.$$

Sketch a graph of T. What is $\lim_{x \to 5} T(x)$? What is $\lim_{x \to 3} T(x)$? Support your assertions.

b) An evil interstellar visitor *changes* exactly *one million values* of S and creates a new function, V. What can be said about $\lim_{x \to a} V(x)$ for all values of a? Support your assertions.

2. Use your calculator to try to figure out what might be the value of

$$\lim_{x \to 0} \frac{(\cot x)(1 - \cos 2x)}{x}$$

by tracing the graph of $\frac{(\cot x)(1-\cos 2x)}{x}$ for x near 0. Show the graph. Then find the exact value of the limit by using a computation based on trig formulas and the fact that $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$.

3. $f(x)$ is a piecewise function defined as follows: $f(x) = \begin{cases} 2x^2 + 2, & \text{if } x < 1 \\ ax^2 + bx, & \text{if } 1 \leq x \leq 2 \\ 2 - \dfrac{6}{x}, & \text{if } x > 2 \end{cases}.$

a) Suppose that $a = 2$ and $b = -3$. Graph $f(x)$ for $0 \leq x \leq 3$. Find the left and right hand limits of $f(x)$ as x approaches 1 and as x approaches 2.

b) Find a and b so that the graph of $f(x)$ doesn't have any jumps (that is, $f(x)$ is continuous everywhere). Graph the resulting function $f(x)$ for $0 \leq x \leq 3$.

4. About a decade ago three centuries of effort by mathematicians culminated in a proof that there were no solutions to the Fermat equation $a^n + b^n = c^n$ if a, b, c, and n are positive integers, with $n > 2$. There are, of course, solutions when $n = 2$: for example, $3^2 + 4^2 = 5^2$.

a) Does the equation $4^x + 5^x = 6^x$ have any solution? (The word "integer" does not appear in the preceding sentence!) If there is a solution, find an approximate value of this solution with accuracy $\pm.05$. If there is no solution, explain why.

b) Suppose a, b, and c are positive real numbers. Explore whether the equation

$$a^x + b^x = c^x$$

must have a solution. This is a "free form" question: try to answer it as well as you can. You are not asked to provide a "formula" for x. You are asked to find conditions which will guarantee that such an x either does or does not exist.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 151 webpage to learn which problem to hand in.