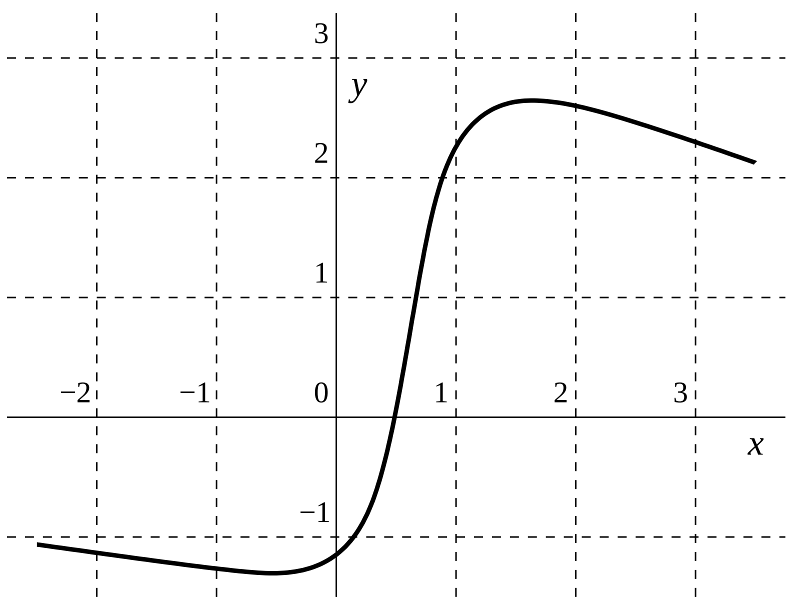
1. Suppose you know that $f'(x) = \frac{2}{1+x^4} - \frac{3}{4+x^4}$. Is $f(0) < f(1)$?

Note You probably can't write a formula for a function with this derivative at this time. Here is such a function (really!):

$$f(x) = \frac{\sqrt{2}}{4}\ln\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right) + \frac{\sqrt{2}}{2}\arctan(\sqrt{2}x+1) + \frac{\sqrt{2}}{2}\arctan(\sqrt{2}x-1) +$$
$$\frac{3}{16}\ln(x^2-2x+2) - \frac{3}{8}\arctan(x-1) - \frac{3}{16}\ln(x^2+2x+2) - \frac{3}{8}\arctan(x+1)\,.$$

I don't think that knowing this formula helps much. Please make an *indirect* argument, using information about f'.

2. A graph of the derivative of $f(x)$ is displayed below. Information about the function $f(x)$ is known only for $-2.5 < x < 3.5$. Also $f(-2) = 1$. Consider the graph carefully, and consider the information in both the numbers and the shapes of the graph (both "quantitative" and "qualitative" information)!



Graph of $y = f'(x)$, the derivative of $f(x)$

a) Explain why $-2 < f(0) < -1$. Look carefully at the graph and make estimates using the MVT. Explain the steps of your reasoning in detail.

b) Explain why $f(3) > 4 + f(1)$. Again, use the MVT and explain your reasoning in detail.

c) How big and how small can $f(1) - f(0)$ be?

d) Use the information in a), b), and c) to explain why $f(3)$ must be positive.

e) Explain why $f(x) = 0$ must have a solution between 0 and 3. Use the IVT and the information obtained in previous parts of this problem.

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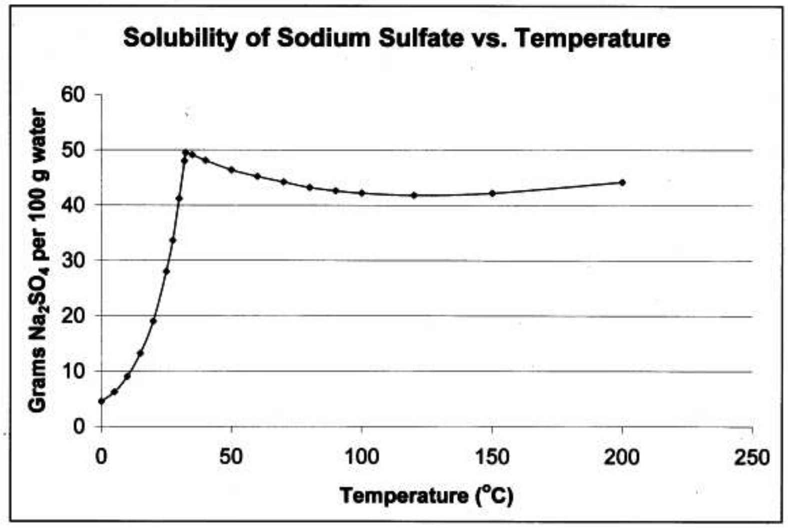
3. The amount of a substance which can be dissolved in a solution may vary with temperature. Below is a graph of the solubility (the maximum amount of the substance) in grams of sodium sulfate, Na_2SO_4, which can be dissolved in 100 grams of water as a function of temperature in degrees Celsius. Suppose $S(T)$ is the solubility at temperature T. Use the graph to answer the following questions as well as you can.



a) Where is $S(T)$ continuous? Where is $S(T)$ differentiable?

b) Where is $S(T)$ increasing? Where is it decreasing? Does $S(T)$ have any local extrema? If yes, where and what type?

c) In what intervals is $S(T)$ concave up? In what intervals is $S(T)$ concave down? Does $S(T)$ have any points of inflection?

d) Sketch a graph of $S'(T)$. What are the units on each axis of your graph?

4 a) Suppose $f(x) = \frac{e^{10x}}{1+e^{10x}}$. Graph this function when $-5 \le x \le 5$, and find the notable features of this graph, including any local extrema, points of inflection, and asymptotes. Sketch a plausible graph of $\frac{e^{10,000x}}{1+e^{10,000x}}$

b) Suppose $g(x) = \frac{x^{10}}{1+x^{10}}$. Graph this function for $-5 \le x \le 5$, and find the notable features of this graph, including any local extrema, points of inflection, and asymptotes. Sketch a plausible graph of $\frac{x^{10,000}}{1+x^{10,000}}$

Note Such functions may serve as appropriate models for biophysical phenomena where rate constants in reactions are very different from everyday time scales. The curves sketched in a) are called *logistic curves.*

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 151 webpage to learn which problem to hand in.