Some answers to the quiz for 152:5-7 and 9-11 3/1/2007

1. The integral $\displaystyle\int_0^2 \frac{1}{\sqrt{2-x}}\,dx$ is an improper integral.
You <u>must</u> analyze this improper integral using the following sequence of steps:

First Write this integral as a limit of proper definite integrals with a varying parameter.

Answer The interval is finite, but the function's domain does *not* include 2, which is an endpoint of the interval. Therefore we should make the definite integral range from 0 to a number less than 2. Here is an acceptable answer:

$$\int_0^2 \frac{1}{\sqrt{2-x}}\,dx = \lim_{A\to 2^-}\int_0^A \frac{1}{\sqrt{2-x}}\,dx$$

Second Evaluate the definite integral with a parameter which appears inside the limit (neither the word "limit" nor the term $\lim_{\to}$ should appear in this stage). Your answer should include one or more expressions with the parameter.

$$\int_0^A \frac{1}{\sqrt{2-x}}\,dx = \int_0^A (2-x)^{-\frac{1}{2}}\,dx = -2\,(2-x)^{\frac{1}{2}}\Big]_0^A =$$
$$-2\sqrt{2-A} - \left(-2\sqrt{2-0}\right) = -2\sqrt{2-A} + 2\sqrt{2}$$

Third Use the previously computed answer and the limit expression you got in the first part of this problem to decide if the improper integral converges, and, if it does, find the value of the integral.

$$\lim_{A\to 2^-}\int_0^A \frac{1}{\sqrt{2-x}}\,dx = \lim_{A\to 2^-} -2\sqrt{2-A} + 2\sqrt{2} = 2\sqrt{0} + 2\sqrt{2} = 2\sqrt{2}$$

2. <u>The horizontal and vertical axes on the graph below **[LEFT BELOW]** have different scales.</u> The graph is a direction field for the differential equation

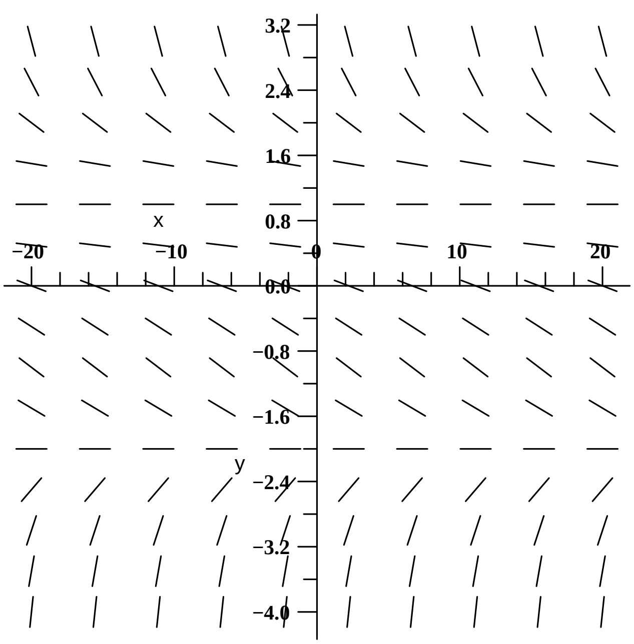
$$y' = -\frac{1}{30}(y-1)^2(2+y)\,.$$

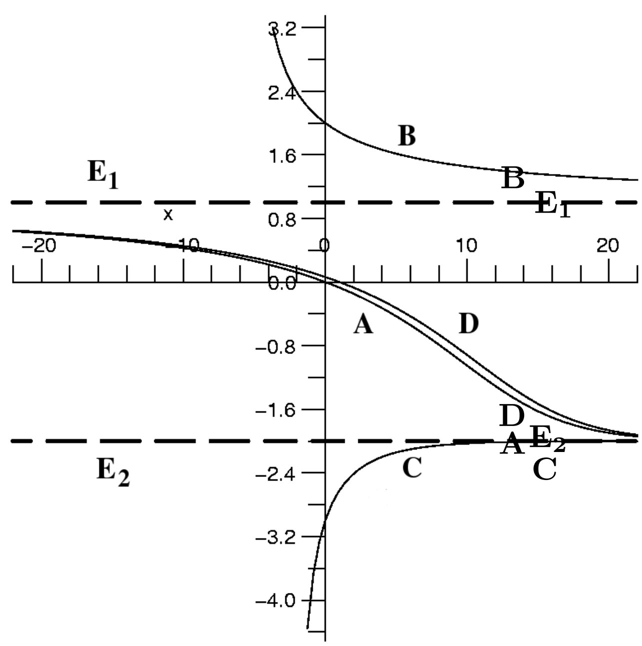
a) Find the equilibrium solutions (where y doesn't change) for this differential equation.

Answer Here $y'=0$ always so y must be constant. The solutions are $y(x)=1$ (which I call $\mathbf{E_1}$) and $y(x)=-2$ (which I call $\mathbf{E_2}$). **[RIGHT BELOW]**

b) Sketch solution curves on the axis above **[RIGHT BELOW]** through these points. Find the indicated limits:

- $(0,0)$. Label this curve **A**. On curve **A**, $\lim_{x\to\infty} y(x) =$ ___– 2___.
- $(0,2)$. Label this curve **B**. On curve **B**, $\lim_{x\to\infty} y(x) =$ ___1___.
- $(0,-3)$. Label this curve **C**. On curve **C**, $\lim_{x\to\infty} y(x) =$ ___– 2___.
- $(1,0)$. Label this curve **D**. On curve **D**, $\lim_{x\to\infty} y(x) =$ ___– 2___.





Comment Both of the pictures shown were drawn by "machine". I don't think it would be very easy to get formulas for the solution curves, so the curves were drawn using a sophisticated numerical approximation program. One surprise was how close the curves **A** and **D** appeared. In restrospect, the graphs seem to almost touch because of the difference in the horizontal and vertical scales.

c) One of the equilibrium solutions is a *stable* equilibrium. Which one?

Answer $\mathbf{E_2}$ or $y=-2$.