Please hand in solutions to these problems on 3/22/2007. Also work on the textbook problems in the syllabus. They certainly will continue to be a major source of exam problems.

Using an integral to estimate an infinite tail

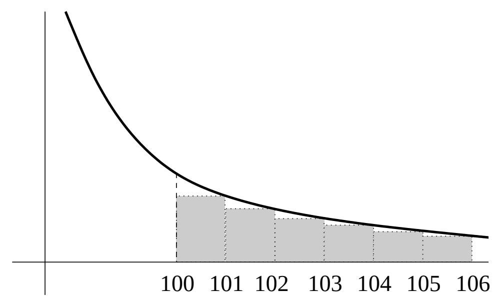
1. A computer reports the following information:

$$\sum_{j=1}^{10} \frac{1}{j^3+2j^2+j} \approx 0.35105; \qquad \sum_{j=1}^{100} \frac{1}{j^3+2j^2+j} \approx 0.35501; \qquad \sum_{j=1}^{1000} \frac{1}{j^3+2j^2+j} \approx 0.35506.$$

This suggests that $\displaystyle\sum_{j=1}^{\infty} \frac{1}{j^3+2j^2+j}$ converges and that its value (to at least 3 decimal places) is 0.355. The following steps show why this is true. Notice that the series has all positive terms, and therefore if you know that the "infinite tail" $\displaystyle\sum_{j=101}^{\infty} \frac{1}{j^3+2j^2+j}$ converges and has sum less than .001, the omitted terms after the first 100 of the whole series won't matter to 3 decimal places. The numerical evidence above is enough.

What the student should do

Overestimate the infinite tail $\displaystyle\sum_{j=101}^{\infty} \frac{1}{j^3+2j^2+j}$ by the infinite tail of a simpler series. Then compare the infinite tail of this simpler series to a simple improper integral. Compute the improper integral. Use this to finish the problem. Hand in also a picture resembling what is drawn to the right, except that that *your* picture should have explicit labels identifying the function whose graph is drawn, and the heights of the boxes shown.



Using a geometric series to estimate an infinite tail

2. A computer reports the following information:

$$\sum_{j=1}^{10} \frac{30^j}{(j!)^2} \approx 6963.86479; \qquad \sum_{j=1}^{15} \frac{30^j}{(j!)^2} \approx 6977.78140; \qquad \sum_{j=1}^{20} \frac{30^j}{(j!)^2} \approx 6977.78249.$$

This suggests that $\displaystyle\sum_{j=1}^{\infty} \frac{30^j}{(j!)^2}$ converges and that its value (to at least 2 decimal places) is 6977.78. The following steps show why this is true. Notice that the series has all positive terms, and therefore if you know that the "infinite tail" $\displaystyle\sum_{j=16}^{\infty} \frac{30^j}{(j!)^2}$ converges and has sum less than .002, the omitted terms after the first 15 of the whole series won't matter to 2 decimal places. The numerical evidence above is enough. In what follows, $a_j = \dfrac{30^j}{(j!)^2}$.

What the student should do

a) If j is a positive integer, simplify the algebraic expression $\dfrac{a_{j+1}}{a_j}$. The result won't be complicated.

b) Use the result from a) to show that if $j \geq 16$, then $\dfrac{a_{j+1}}{a_j} < 0.11$. (Show all steps. You'll need a calculator!)

c) You may assume (this is true!) that $a_{16} \approx .000983$. Use this fact and what was done in b) to compare $\displaystyle\sum_{j=16}^{\infty} a_j$ to a geometric series, each of whose terms is individually larger than this series. Find the sum of the geometric series, which should be less than .002, so that the omitted infinite tail of the original series is small enough. (Show all steps. You'll need a calculator!)