Please hand in solutions to these problems on 3/8/2007. Also work on the textbook problems in the syllabus. They certainly will continue to be a major source of exam problems.

Separable Differential Equations

1. Consider the differential equations

a) $\dfrac{dy}{dx} = 2x + 3y$ b) $\dfrac{dy}{dx} = e^{2x+3y}$ c) $\dfrac{dy}{dx} = x^3y^2$ d) $\dfrac{dy}{dx} = x^2 + y^3$

Two of these are separable. For each of these two separable equations, solve the initial value problem whose initial condition is $y(0) = 1$. In each case your solution should be $y = f(x)$ where $f(x)$ is a formula.

Sequences

2. Write decimal approximations for the first 5 terms of the sequence $a_n = \frac{20^n}{n!}$, beginning with $n = 1$. It is true that $\lim_{n\to\infty} a_n = 0$. Briefly explain why. (Suggestion: think about how the terms change when n is larger than 40 – do they grow or shrink? How much?)

3. Suppose $f(x) = \sqrt{2+3x}$, and that a sequence $\{a_n\}$ is defined by the following recursive process:

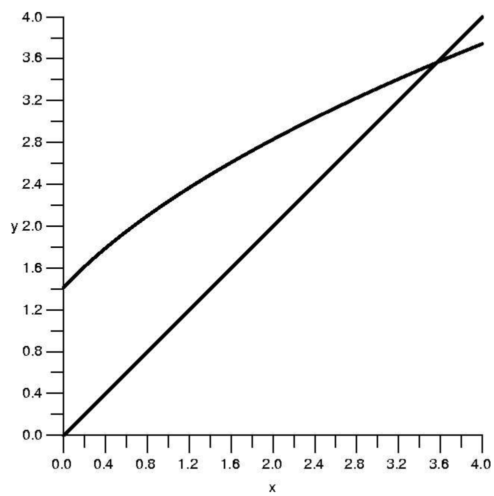
$a_1 = 1$; $a_{n+1} = f(a_n)$ for $n > 1$.

a) Compute decimal approximations for the first 5 terms, a_1, a_2, a_3, a_4, and a_5, of the sequence.

b) The graph to the right shows parts of the line $y = x$ and the curve $y = \sqrt{2+3x}$. Locate on this graph or on a copy to be handed in the following points: (a_1, a_2), (a_2, a_2), (a_2, a_3), (a_3, a_3), (a_3, a_4), (a_4, a_4), (a_4, a_5), and (a_5, a_5). Also show a_1, a_2, a_3, a_4, and a_5 on the $x-$axis. (You must draw **13 points**.)

c) Write a statement of a result from section 11.1 which shows that this sequence converges.

d) Compute the limit of $\{a_n\}$.



Series

4. A 1×1 square is "dissected" by three equally spaced horizontal lines and by three equally spaced vertical lines. The central square is shaded. Then the bordering Northeast, Northwest, Southeast, and Southwest squares are similarly dissected, with the central square shaded. Each of *those* dissected squares has something similarly done to their borders, etc. The diagram to the right shows this process only for the first three steps.

a) How many new shaded squares are introduced at the n^{th} step? (There is one shaded square at the first step.) What is the side length of the squares which are introduced at the n^{th} step?

b) What is the total sum, as n goes from 1 to ∞, of the shaded area (all the shaded squares)?