Homework Problems for Chapter 11, Section 10

(1) Find the 3rd-degree Taylor polynomial $T_3(x)$ of $f(x) = \sin x$ centered at $a = \pi/6$.

(2) Find the Maclaurin series of $\cos^2(x/4)$ using the Maclaurin series of $\cos x$ and a trigonometric identity.

(3) Find a reasonable estimate for $|\sin(10^{-2}) - (10^{-2} - 10^{-6}/6)|$. Hint: $10^{-2} - 10^{-6}/6 = T_4(10^{-2})$.

(4) Find the Maclaurin series for $f(x) = \dfrac{e^{2x} - 1 - 2x}{x^2}$ using the Maclaurin series for e^x.

(5) Use $\tan x = x + (1/3)x^3 + (2/15)x^5 + \cdots$ and the Maclaurin series for $\cos x$ to compute the first three nonzero terms in the Maclaurin series for $(\tan x)(\cos x)$. Since $(\tan x)(\cos x) = \sin x$, you know what the answer should be. This is a way to check the arithmetic in the long division that led to $\tan x = x + (1/3)x^3 + (2/15)x^5 + \cdots$.

(6) Find $\displaystyle\lim_{x \to 0} \dfrac{x + (1/3)x^3 - \tan x}{x^5}$ using a simple method that does not involve l'Hôpital's Rule.