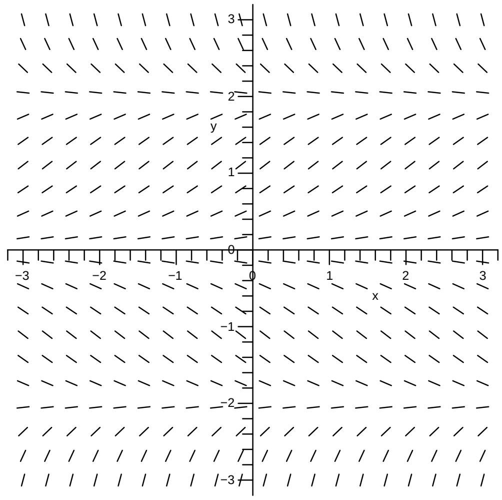
(12) 1. The graph is a direction field for the differential equation $y' = y\left(1 - \frac{1}{4}y^2\right)$.



a) Find the equilibrium solutions (where y doesn't change) for this differential equation.

b) Sketch solution curves on the graph above through the points below and find the indicated limits:

- $(0,1)$. Label this curve **A**. On curve **A**, $\lim_{x\to-\infty} y(x)$ = ______ and $\lim_{x\to+\infty} y(x) =$ ______.
- $(0,-1)$. Label this curve **B**. On curve **B**, $\lim_{x\to-\infty} y(x)$ = ______ and $\lim_{x\to+\infty} y(x) =$ ______.

c) One of the equilibrium solutions is *not* stable as $x \to +\infty$. Which is that solution?

(14) 2. Find the solution of the differential equation $y' = \frac{xy}{\ln y}$ which satisfies the initial condition $y(0) = 3$. In the answer express y explicitly as a function of x.

(12) 3. The series $\sum_{n=1}^{\infty} \frac{1}{7\sqrt{n}+2^n}$ converges and its sum, to an accuracy of .001, is .314. Find a positive integer N so that the partial sum, $S_N = \sum_{n=1}^{N} \frac{1}{7\sqrt{n}+2^n}$, has a value within .001 of the sum of the whole series. Explain your reasoning.

Comment You are *not* asked to find the "best possible" N, only to find a convenient N which satisfies the requirement and to support your assertion.

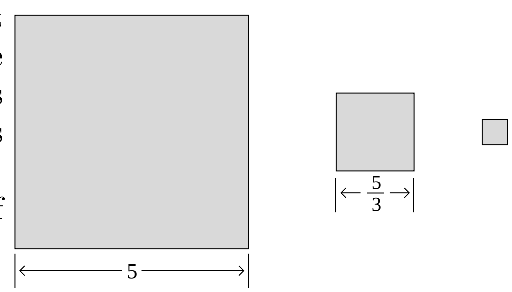
n	2^n
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1,024
11	2,048
12	4,096

(14) 4. This problem is about the power series $\sum_{n=1}^{\infty} \left(\frac{3n+2}{n^2}\right) x^n$.

a) What is the radius of convergence of this power series?

b) What is the behavior of the power series (divergence, absolute or conditional convergence) on the boundary points of the interval of convergence?

(12) 5. An infinite sequence of squares is drawn. The first three are shown below. The first square has side length 5, and each square after the first square has side length equal to one-third of the preceding square's side length.



a) What is the total length of the perimeter of all of the squares?

b) What is the total area inside all of the squares?

(12) 6. The infinite series $\sum_{n=1}^{\infty} \frac{4}{n^{1/3}}$ diverges. Find N so that the partial sum, $\sum_{n=1}^{N} \frac{4}{n^{1/3}}$, is larger than 100.
Comment You are *not* asked to find the "best possible" N, only to find a convenient N which satisfies the requirement and to support your assertion.

(12) 7. a) Suppose the sequence $\{a_n\}$ is defined by $a_n = \left(1 + \frac{3}{n}\right)^{2n}$. Find the exact value of the limit of this sequence.
b) Assume that the numbers b_n are defined recursively by $b_1 = 1$, $b_{n+1} = \sqrt{5 + 2b_n}$ for $n = 1, 2, 3, \ldots$. Assume also that we have already shown that the sequence $\{b_n\}$ converges. Find $\lim_{n\to\infty} b_n$.

(12) 8. a) Find the power series for $\frac{1}{2+x^3}$ centered at $x = 0$. Either write at least the first 4 non-zero terms of the series, or write the whole series in summation form.
Hint Geometric!
b) Find the first 6 non-zero terms of the power series for $\frac{5-7x}{2+x^3}$ centered at $x = 0$. The answer should be a polynomial in x with exactly 6 terms of different degrees in x and with non-zero coefficients.
Hint Use the answer to a).

A A

Second Exam for Math 152

Sections 5, 6, 7 and 9, 10, 11

April 11, 2007

NAME ____

SECTION __________

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.

No texts, notes, or calculators other than the attached formula sheet may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	14	
3	12	
4	14	
5	12	
6	12	
7	12	
8	12	
Total Points Earned:		

A A