Formula sheets for the Math 152 Final Exam

The solutions of $ax^2+bx+c=0$ are $x=\big(-b\pm\sqrt{b^2-4ac}\big)/(2a)$.

$$e^{a+b}=e^a e^b\ ,\quad \ln(ab)=(\ln a)+(\ln b)\ ,\quad \ln(a^b)=b(\ln a)\ ,\quad \ln(1)=0\ ,\quad \ln(e)=1$$

$$e^{\ln x}=x\ ,\quad \ln(e^x)=x\ ,\quad \tfrac{d}{dx}(e^x)=e^x\ ,\quad \tfrac{d}{dx}(\ln x)=1/x\ ,\quad \int \tfrac{du}{u}=\ln|u|+C$$

$$\cos^2 x+\sin^2 x=1\ ,\quad 1+\tan^2 x=\sec^2 x\ ,\quad 1+\cot^2 x=\csc^2 x$$

$$\sin(x+y)=\sin x\cos y+\cos x\sin y\ ,\quad \cos(x+y)=\cos x\cos y-\sin x\sin y$$

$$\sin(2x)=2\sin x\cos x\ ,\quad \cos(2x)=\cos^2 x-\sin^2 x$$

$$\cos^2 x=\frac{1+\cos(2x)}{2}\ ,\quad \sin^2 x=\frac{1-\cos(2x)}{2}$$

$$\frac{d}{dx}(\sin x)=\cos x\ ,\quad \frac{d}{dx}(\tan x)=\sec^2 x\ ,\quad \frac{d}{dx}(\sec x)=\sec x\tan x$$

$$\frac{d}{dx}(\cos x)=-\sin x\ ,\quad \frac{d}{dx}(\cot x)=-\csc^2 x\ ,\quad \frac{d}{dx}(\csc x)=-\csc x\cot x$$

$$\frac{d}{dx}(\sin^{-1} x)=\frac{1}{\sqrt{1-x^2}}\ ,\quad \frac{d}{dx}(\tan^{-1} x)=\frac{1}{1+x^2}\ ,\quad \frac{d}{dx}(\sec^{-1} x)=\frac{1}{x\sqrt{x^2-1}}$$

$$\int \sec x\,dx=\ln|\sec x+\tan x|+C\ ,\quad \int \csc x\,dx=\ln|\csc x-\cot x|+C$$

The area between concentric circles is $\pi(\text{outer radius})^2-\pi(\text{inner radius})^2$.

The area of a cylinder is $(2\pi\,\text{radius})(\text{height})$.

If the force is constant then work = force $\times$ distance.

The average value of f on $[a,b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$.

If a proper rational function has $(ax+b)^r$ in the denominator, then its partial fraction expansion must include the sum $\dfrac{A_1}{ax+b}+\dfrac{A_2}{(ax+b)^2}+\cdots+\dfrac{A_r}{(ax+b)^r}$. If ax^2+bx+c is irreducible and a proper rational function has $(ax^2+bx+c)^r$ in the denominator, then its partial fraction expansion must include the sum

$$\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\frac{A_rx+B_r}{(ax^2+bx+c)^r}.$$

Midpoint Rule: $\Delta x[f(\overline{x}_1)+f(\overline{x}_2)+\cdots+f(\overline{x}_n)]$ where $\overline{x}_i=(x_{i-1}+x_i)/2$. Trapezoidal Rule: $\dfrac{\Delta x}{2}[f(x_0)+2f(x_1)+2f(x_2)+\cdots+2f(x_{n-2})+2f(x_{n-1})+f(x_n)]$. Simpson's Rule: $\dfrac{\Delta x}{3}[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+\cdots+2f(x_{n-2})+4f(x_{n-1})+f(x_n)]$.

If E_T and E_M are the errors for the Trapezoidal Rule and Midpoint Rule, respectively, then $|E_T|\le\dfrac{K(b-a)^3}{12n^2}$ and $|E_M|\le\dfrac{K(b-a)^3}{24n^2}$, where $|f''(x)|\le K$ for $a\le x\le b$.

$$|\text{Error in Simpson's Rule}|\le\frac{K(b-a)^5}{180n^4}\ \text{where}\ |f^{(4)}(x)|\le K\ \text{for}\ a\le x\le b.$$

$$\text{length}=\int_a^b\sqrt{1+[f'(x)]^2}\,dx\ ,\quad \text{surface area}=\int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx$$

$$\lim_{n\to\infty} n^{1/n} = 1 \ ; \quad \lim_{n\to\infty} \frac{x^n}{n!} = 0 \ ; \quad \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \ ; \quad \lim_{n\to\infty} \frac{n^k}{a^n} = 0 \text{ if } a > 1.$$

$$\lim_{n\to\infty} c^n = 0 \text{ if } |c| < 1 \ ; \quad \sum_{n=0}^{\infty} c^n = \frac{1}{1-c} \text{ if } |c| < 1.$$

$$\sum_{n=1}^{\infty} \frac{1}{n^p} \text{ converges if } p > 1 \text{ (and diverges if } p \le 1).$$

If the statement $\lim_{n\to\infty} a_n = 0$ is false, then $\sum_{n=1}^{\infty} a_n$ diverges.

If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
If $0 \le a_n \le b_n$ and $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

If $0 < a_n$, $0 < b_n$, $0 < \lim_{n\to\infty} \frac{a_n}{b_n} < \infty$, then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge.

If $f(x)$ is a positive decreasing continuous function and $a_n = f(n)$ then
$\int_{n+1}^{\infty} f(x)\,dx \le a_{n+1} + a_{n+2} + a_{n+3} + \cdots \le \int_{n}^{\infty} f(x)\,dx.$

If $a_n > 0$, $a_1 \ge a_2 \ge a_3 \ge \cdots$ and $\lim_{n\to\infty} a_n = 0$ then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

$\sum a_n$ converges absolutely when $\sum |a_n|$ converges. $\sum a_n$ converges conditionally when it converges, but does not converge absolutely. If $\sum a_n$ converges absolutely, then $\sum a_n$ converges.

$$\text{If } a_n \ne 0 \text{ and } \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = L \text{ then } \begin{cases} \sum a_n \text{ converges absolutely if } L < 1, \\ \sum a_n \text{ diverges if } L > 1, \\ \text{the test is inconclusive if } L = 1. \end{cases}$$

$$\text{If } \lim_{n\to\infty} |a_n|^{1/n} = L \text{ then } \begin{cases} \sum a_n \text{ converges absolutely if } L < 1, \\ \sum a_n \text{ diverges if } L > 1, \\ \text{the test is inconclusive if } L = 1. \end{cases}$$

The nth Taylor polynomial of $f(x)$ with center a is $T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k$. The Taylor series of $f(x)$ with center a is $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$. $R_n(x) = f(x) - T_n(x)$.

If $|f^{(n+1)}(x)| \le M$ for $|x-a| \le d$, then $|R_n(x)| \le \frac{M}{(n+1)!}|x-a|^{n+1}$ for $|x-a| \le d$.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \ ; \quad \sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \ ; \quad \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \ ;$$

$$(1+x)^k = 1 + \sum_{n=1}^{\infty} \left(\frac{k(k-1)(k-2)\cdots(k-n+1)}{n!} \right) x^n \text{ if } |x| < 1.$$

$$\text{length} = \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt, \quad \text{length} = \int_{a}^{b} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta, \quad \text{area} = \int_{a}^{b} \frac{r^2}{2}\, d\theta.$$